Characteristics of soliton propagation in an isotropic chiral metamaterial

N. L. Tsitsas[1], A. Lakhtakia[2], and D. J. Frantzeskakis[3]

[1]Department of Informatics, Aristotle University of Thessaloniki
GR-54124, Thessaloniki, Greece
Fax: + 30-2310998419; email: ntsitsas@csd.auth.gr
[2]Department of Engineering Science and Mechanics
Pennsylvania State University
PA 16802-6812, University Park, USA
email: akhlesh@psu.edu
[3]Department of Physics, University of Athens
GR-15784, Athens, Greece
email: dfrantz@phys.uoa.gr

Abstract

By using the reductive perturbation method, we found that the two Beltrami components of the electromagnetic field propagating along a fixed direction in an isotropic nonlinear chiral metamaterial obey a system of two coupled nonlinear Schrödinger (NLS) equations. Certain spectral regimes exist wherein one of the two Beltrami components exhibits a negative-real refractive index and the NLS system can be approximated by the Manakov system. Bright-bright, dark-dark and dark-bright vector solitons can form in those spectral regimes.

1. Introduction

Metamaterials exhibiting negative-real refractive index (NRRI) have been a topic of intense research activity [1]. A sufficient condition for a linear isotropic material to exhibit an NRRI is that its permittivity and permeability have negative real parts in the same frequency interval [2]. That condition can be relaxed if the material possesses chirality and, therefore, exhibits circular birefringence; then, one of the two refractive indices can have a negative real part [3]. This possibility of linear isotropic chiral NRRI materials gave fresh impetus to chirality research [4]-[5]. On the other hand, nonlinear isotropic achiral NRRI materials [6] may feature nonlinearity-induced localization of electromagnetic (EM) waves and soliton formation [7]. The possible incorporation of nonlinearity in isotropic chiral NRRI materials motivated us to investigate soliton formation in these materials.

We studied the propagation of an EM field along a fixed direction in an isotropic chiral NRRI material with nonlinear permittivity and permeability of the Kerr type. We started from the time-domain Maxwell equations and used the reductive perturbation method (RPM) to derive a system of two coupled nonlinear Schrödinger (NLS) equations for the left-handed and right-handed Beltrami components of the EM field. Then, we adopted the Lorentz model for the linear parts of the permittivity and permeability and the Condon model for the chirality parameter [8]. For sufficiently large chirality parameter in a certain spectral regime, the refractive index for the left/right-handed Beltrami component is real and negative while that for the right/left-handed Beltrami component is real and positive; moreover, the system of the NLS equations can be approximated by the completely integrable Manakov system [9]. Hence, the nonlinear isotropic chiral NRRI material can support various classes of exact vector soliton solutions: bright-bright solitons as well as dark-dark and dark-bright solitons. In all cases, these vector solitons are composed of a Beltrami component with negative- and another with positive-real refractive index.

2. Coupled nonlinear Schrödinger equations

We considered an isotropic chiral material obeying the Tellegen constitutive relations [10], and possessing the following permittivity and the permeability functions (in the frequency domain):

$$\tilde{\epsilon}(\omega)=\tilde{\epsilon}_R(\omega)+i\tilde{\epsilon}_I(\omega)+\alpha_1|\mathbf{E}|^2+\beta_1|\mathbf{H}|^2,\quad \tilde{\mu}(\omega)=\tilde{\mu}_R(\omega)+i\tilde{\mu}_I(\omega)+\alpha_2|\mathbf{E}|^2+\beta_2|\mathbf{H}|^2. \tag{1}$$

For both functions the nonlinear parts are characterized by a Kerr nonlinearity, with $\alpha_{1,2}$ and $\beta_{1,2}$ the Kerr coefficients. The chirality parameter is purely linear and is defined as $\tilde{\kappa}(\omega)=\tilde{\kappa}_R(\omega)+i\tilde{\kappa}_I(\omega)$.

We assumed next that the EM field is propagating along the $+z$ axis with a carrier frequency ω_c. Then, the Bohren decomposition into left- (+) and right- ($-$) handed Beltrami components implies [10]:

$$\mathbf{E}(z,t)=\left[\hat{\mathbf{e}}_+q^+(z,t)\mathrm{e}^{ik_c^+z}+\hat{\mathbf{e}}_-q^-(z,t)\mathrm{e}^{ik_c^-z}\right]\mathrm{e}^{-i\omega_c t}+\text{c.c.}, \tag{2}$$

$$\mathbf{H}(z,t)=\left[\hat{\mathbf{e}}_+p^+(z,t)\mathrm{e}^{ik_c^+z}+\hat{\mathbf{e}}_-p^-(z,t)\mathrm{e}^{ik_c^-z}\right]\mathrm{e}^{-i\omega_c t}+\text{c.c.}, \tag{3}$$

where "c.c." denotes the complex conjugate, $k_c^{\pm}=\tilde{k}^{\pm}(\omega_c)$, $\hat{\mathbf{e}}_{\pm}=(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})/\sqrt{2}$, while the field envelopes $q^{\pm}$ and $p^{\pm}$ have to be determined; also $k_c^{\pm(1)}=d\tilde{k}^{\pm}(\omega)/d\omega|_{\omega=\omega_c}$ and $k_c^{\pm(2)}=d^2\tilde{k}^{\pm}(\omega)/d\omega^2|_{\omega=\omega_c}$, where $\tilde{k}^{\pm}(\omega)$ and $\tilde{n}^{\pm}(\omega)$ are the complex wave numbers and refractive indices, respectively.

Nonlinear evolution equations for the field envelopes were found by the reductive perturbation method, as described in [11]. First, we introduced the slow variables $Z=\varepsilon^2 z$ and $T^{\pm}=\varepsilon(t-k_c^{\pm(1)}z)$, and then expanded $q^{\pm}$ and $p^{\pm}$ in terms of the formal small parameter ε. Next, we substituted the latter into the Maxwell equations, and expanded the linear parts of $\tilde{\epsilon}(\omega)$, $\tilde{\mu}(\omega)$, and $\tilde{\kappa}(\omega)$ about ω_c. Assuming that $\alpha_{1,2},\beta_{1,2},\tilde{\epsilon}_I,\tilde{\mu}_I,\tilde{\kappa}_I$ are of $\mathcal{O}(\varepsilon^2)$, we obtained at $\mathcal{O}(\varepsilon^2)$ the coupled (dimensionless) NLS equations:

$$i\partial_Z\phi^+-\tfrac{s}{2}\partial_T^2\phi^++\sigma\left(|\phi^+|^2+|\phi^-|^2\right)\phi^+=-i\Gamma^+\phi^+, \tag{4}$$

$$i\left(\partial_Z\phi^--\delta\,\partial_T\phi^-\right)-\tfrac{d}{2}\partial_T^2\phi^-+\sigma\left(|\phi^+|^2+|\phi^-|^2\right)\phi^-=-i\Gamma^-\phi^-, \tag{5}$$

where the independent variables are $T=(t-k_c^{+(1)}z)/t_0$ and Z/L^+ ($L^+=t_0^2/|k_c^{+(2)}|$, t_0 a characteristic time, and c the speed of light in free space), while the constant parameters in Eqs. (4)-(5) read:

$$\delta=t_0\frac{k_c^{+(1)}-k_c^{-(1)}}{|k_c^{+(2)}|},\quad d=\frac{k_c^{-(2)}}{|k_c^{+(2)}|},\quad s=\frac{k_c^{+(2)}}{|k_c^{+(2)}|},\quad \sigma=\gamma/|\gamma|,\quad \Gamma^{\pm}=L^+\tilde{\Gamma}^{\pm}, \tag{6}$$

$$\gamma=\frac{\omega_c}{2}[\epsilon_0(\alpha_1\tilde{Z}_\ell+\beta_1\tilde{Z}_\ell^{-1})+\mu_0(\alpha_2\tilde{Z}_\ell+\beta_2\tilde{Z}_\ell^{-3})],\quad \tilde{\Gamma}^{\pm}=\omega_c\left(\frac{\tilde{\epsilon}_I\tilde{\mu}_R+\tilde{\epsilon}_R\tilde{\mu}_I}{2\sqrt{\tilde{\epsilon}_R\tilde{\mu}_R}}\pm\frac{\tilde{\kappa}_I}{c}\right). \tag{7}$$

In the most general case ($\delta\neq 0$, $d\neq\pm 1$ and $\Gamma^{\pm}\neq 0$), the system of Eqs. (4) and (5) is not integrable. However, for certain parameter values, one can find various types of vector solitons [12]. Provided $\delta=0$, $d=s$ and $\Gamma^{\pm}=0$, Eqs. (4) and (5) reduce to the completely integrable Manakov system [9]:

$$i\partial_Z\mathbf{u}-\frac{s}{2}\partial_T^2\mathbf{u}+\sigma|\mathbf{u}|^2\mathbf{u}=\mathbf{0}, \tag{8}$$

where $\mathbf{u}(Z,T)=[\phi^+(Z,T),\phi^-(Z,T)]^T$.

3. Vector solitons

We found that it is possible to obtain physically relevant conditions allowing us to approximate the general system of the NLS Eqs. (4) and (5) to the Manakov system (8). We considered a certain type of isotropic chiral material with single-resonance Lorentz models for the linear parts of the relative permittivity and permeability, and to the Condon model for the chirality [8]:

$$\tilde{\epsilon}_\ell(\omega)=1-\frac{\omega_p^2}{\omega^2-\omega_\epsilon^2+2i\delta_\epsilon\omega},\ \tilde{\mu}_\ell(\omega)=1-\frac{\omega_m^2}{\omega^2-\omega_\mu^2+2i\delta_\mu\omega},\ \tilde{\kappa}(\omega)=\frac{acZ_0^{-1}\omega}{\omega^2-\omega_\kappa^2+2i\delta_\kappa\omega}, \tag{9}$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0}$, $\{\omega_\epsilon, \delta_\epsilon\}$, $\{\omega_\mu, \delta_\mu\}$, and $\{\omega_\kappa, \delta_\kappa\}$ are the resonance frequencies and linewidths of $\tilde{\epsilon}_\ell$, $\tilde{\mu}_\ell$, and $\tilde{\kappa}$, respectively; while a is the rotatory strength of the resonance.

For $\omega_p = 0.9\omega_\epsilon$, $\omega_m = 0.8\omega_\epsilon$, $\omega_\mu = 0.7\omega_\epsilon$, $\omega_\kappa = 0.5\omega_\epsilon$, $\delta_\epsilon = \delta_\mu = \delta_\kappa = 2 \times 10^{-3}\omega_\epsilon$, and $a = 0.5\omega_\epsilon/c$, there exists a certain spectral "Manakov" regime such that $\mathrm{Re}(\tilde{n}^-) < 0$, $\mathrm{Re}(\tilde{n}^+) > 0$, $d \approx s$, while $\delta \approx 0$ and $\Gamma^\pm \approx 0$; see Fig. 1. Thus, Eqs. (4) and (5) are well approximated by the Manakov system (8).

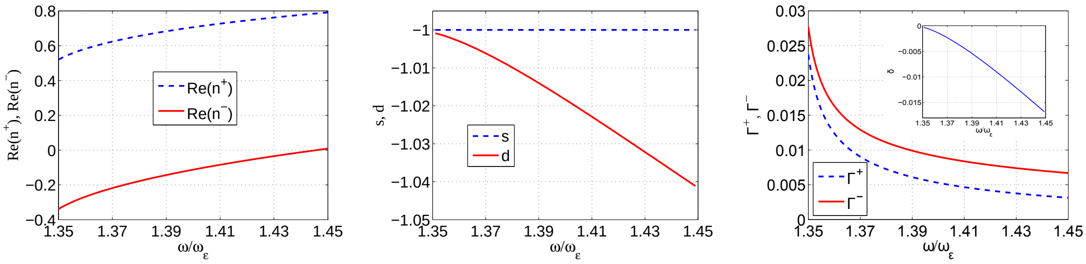

Fig. 1: (a) $\mathrm{Re}(\tilde{n}^-)$ and $\mathrm{Re}(\tilde{n}^+)$ in the Manakov regime $(1.35\omega_\epsilon, 1.45\omega_\epsilon)$, (b) s and d of the NLS Eqs. (4)-(5), (c) Γ^+ and Γ^- in the same regime. Inset shows the coefficient δ.

For $s = d = -1$, we considered $\sigma = \pm 1$ where the effective Kerr nonlinearity in the Manakov system is *self-focusing* and *self-defocusing*, respectively. For $\sigma = +1$ there exist exact bright-bright soliton solutions, i.e., both Beltrami components take the form of bright solitons. For $\sigma = -1$ there exist both dark-dark solitons (both Beltrami components take the form of dark solitons) and dark-bright ones (one Beltrami component is a dark and the other a bright soliton). The form of these solitons is given in [11].

4. Conclusions

We studied pulse propagation in an isotropic chiral NRRI material with Kerr nonlinearity and found that it exhibits a NRRI for the right/left-handed Beltrami component in a certain spectral regime whereas the left/right-handed Beltrami component does not. In that regime, the NLS system may be approximated well by the Manakov system supporting bright-bright, dark-dark or dark-bright vector solitons solutions.

References

[1] S. A. Ramakrishna, Physics of negative refractive index materials, *Rep. Prog. Phys.*, vol. 68, p. 449, 2005.
[2] R. A. Depine and A. Lakhtakia, A new condition to identify isotropic dielectric-magnetic materials displaying negative phase velocity, *Microw. Opt. Technol. Lett.*, vol. 41, p. 315, 2004.
[3] J. B. Pendry, A chiral route to negative refraction, *Science*, vol. 306, p. 1353, 2004.
[4] T. G. Mackay and A. Lakhtakia, Negatively refracting chiral metamaterials: a review, *SPIE Rev.*, vol. 1, 018003, 2010.
[5] S. Tretyakov, A. Sihvola, and L. Jylhä, Backward-wave regime and negative refraction in chiral composites, *Photon. Nanostruct.*, vol. 3, p. 107, 2005.
[6] A. A. Zharov, I. V. Shadrivov, and Yu. S. Kivshar, Nonlinear properties of left-handed metamaterials, *Phys. Rev. Lett.*, vol. 91, 037401, 2003.
[7] N. L. Tsitsas, N. Rompotis, I. Kourakis, P. G. Kevrekidis, and D. J. Frantzeskakis, Higher-order effects and ultrashort solitons in left-handed metamaterials, *Phys. Rev. E*, vol. 79, 037601, 2009.
[8] S. A. Maksimenko, G. Ya. Slepyan, and A. Lakhtakia, Gaussian pulse propagation in a linear, lossy chiral medium, *J. Opt. Soc. Am. A,* vol. 14, p. 894, 1997.
[9] S.V. Manakov, Theory of 2-Dimensional Stationary Self-Focusing of Electromagnetic Waves, *Zh. Eksp. Teor. Fiz.*, vol. 65, p. 505, 1973 [Sov. Phys. JETP, vol. 38, p. 248, 1974].
[10] A. Lakhtakia, *Beltrami Fields in Chiral Media,* Singapore: World Scientific, 1994.
[11] N. L. Tsitsas, A. Lakhtakia, and D. J. Frantzeskakis, Vector solitons in nonlinear isotropic chiral metamaterials, *J. Phys. A,* vol. 44, 435203, 2011.
[12] Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals,* Acad. Press, 2003.

ISBN 978-952-67611-2-1