Enhancing nonlinear response with metamaterials: from nonlinear parameters engineering to one-way imaging

E. Poutrina, A. Rose, and D. R. Smith

Center for Metamaterials and Integrated Plasmonics
Department of Electrical and Computer Engineering
Duke University
P.O. Box 90291, 130 Hudson Hall, Durham, NC, USA
Fax: + 919–6605293; email: ekaterina.poutrina@duke.edu

Abstract

We first summarize the approaches developed in our group allowing for the description of a nonlinear metamaterial media in terms of the effective nonlinear susceptibilities. We subsequently discuss the nonlinear magneto-electric coupling and demonstrate that it can lead to a non-reciprocal nonlinear imaging using a three- or four-wave mixing process, producing an image either on reflection or on transmission.

1. Nonlinear effective parameters—theory versus experiment.

For any metamaterial based on shaped metallic elements incorporated within a dielectric medium, a highly non-uniform field distribution within a metamaterial unit cell leads to the existence of strongly-enhanced field regions within the medium. It has been long predicted [1] that any nonlinear material or element placed in a vicinity of a region of such a field enhancement will exhibit a strong nonlinear response, leading to an enhanced effective nonlinear response of the whole composite medium. Indeed, various nonlinear phenomena such as harmonic generation, tunability, or parametric down-conversion have been demonstrated experimentally [2-4]. An extensive amount of work has also been done on the theoretical analysis of wave propagation through a layer of negative-index medium possessing the assumed linear and nonlinear parameters [5,6]. Such extension of the analysis of the unique electromagnetic properties of metamaterials to the regime of nonlinear wave propagation gives rise to a completely new branch of modern nonlinear optics. A wide class of novel applications ranging from bioimaging and sensors, with improved resolution and sensitivity, to communications and stealth technology can be envisioned.

However, the most success of metamaterials while considering a linear wave propagation regime came from the ability to precisely prescribe the effective electromagnetic parameters characterizing the resulting effective medium. For practical applications, it is extremely desirable to have a similar representation in the nonlinear regime that would allow relating a particular geometry of the unit cell to the resulting effective nonlinear susceptibility of the composite medium.

Several approaches have been developed in our group to accomplish this task. One approach [7] employs a nonlinear oscillator model for describing a response of the unit cell and follows a standard path used in nonlinear optics to derive the nonlinear susceptibilities of the medium by using a series representation of the effective nonlinear magnetization in terms of the strength of the applied field. The second approach [8] uses the transfer matrix method and allows the retrieval of the effective nonlinear susceptibility of the metamaterial medium from the known S- parameters that are obtained either experimentally or from simulations, extending the approach used in the linear case to account for a nonlinear response. Both approaches have been extensively tested experimentally [7-10] em-

ISBN 978-952-67611-2-1

ploying a varactor-loaded split ring resonator (VLSRR) medium as an example and showed an excellent agreement with each other and with the experimental data. One example of testing the experimental values of second harmonic (SH) generation versus the nonlinear oscillator model predictions is shown in Fig. 1 for several values of the excitation power [9].

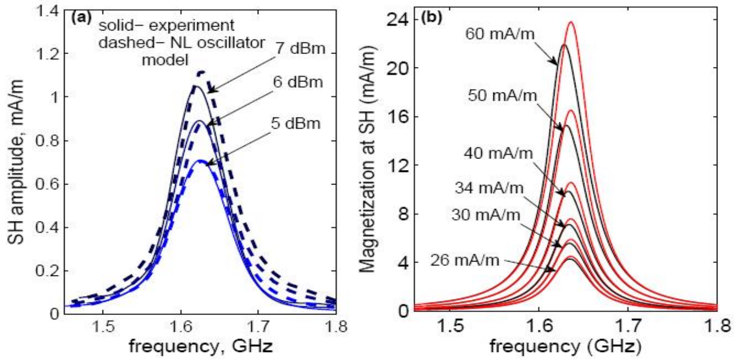

Fig. 1. (a) Comparison of the SH amplitude measured experimentally and obtained from the FEM simulations assuming the nonlinear oscillator model for the effectuve nonlinear magnetization. (b) Theoretically predicted (red) versus obtained from the simlations (black) effective magmetization at the second harmonic frequency.

2. Uni-directional response and non-reciprocal nonlinear imaging.

The above examples assume the presence of a single type of nonlinearity in the medium, e.g. a magnetic effective nonlinear response in case of the employed VLSRR medium. The analysis becomes more involved when several types of nonlinearity are present simultaneously, leading to coupling of the electric and magnetic response in the nonlinear regime. We show that, under certain conditions, such coupling leads to a uni-directional generation of the field produced by the second or the third order nonlinear process.

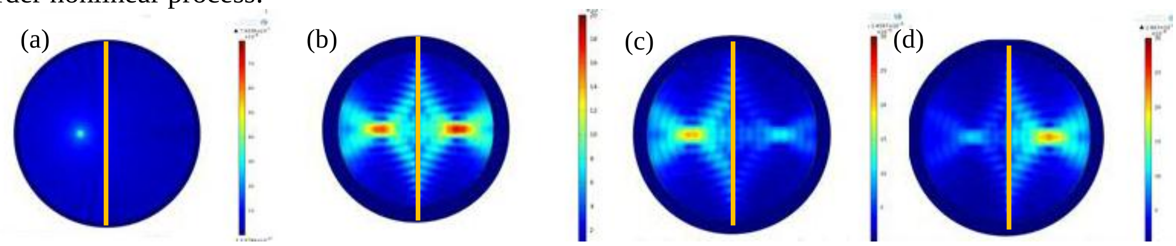

Fig. 2. Nonlinear image generation with TWM. (a) A point source at frequency ω_s located on the left from the thin nonlinear slab (yellow line). A plane wave pump at ω_p is incident from the left. (b) An image in the slab at $\omega_i = \omega_p - \omega_s$, in the case of a purely magnetic nonlinearity in the slab. (c) Same as (b), but with both magnetic and electric nonlinearities present, with nonlinear polarization and nonlinear magnetization oscillating in phase. (d) Same as (c), with nonlinear polarization and nonlinear magnetization oscillating out of phase.

As one application of the above response, we demonstrate a uni-directional imaging employing a process of three-wave mixing (TWM) in a deeply sub-wavelength layer of a metamaterial possessing a magneto-electric nonlinear response, shown in Fig. 2. The phase matching requirements do not come into play in case of such a thin-layer nonlinear medium, such that the direction of the generated by TWM field is mostly governed by the momentum conservation requirements. As a result, in case of either a purely magnetic or purely electric nonlinearity, a pump at frequency ω_p and a source at ω_s produce an image at frequency $\omega_i = \omega_p - \omega_s$ on each side of the film, as shown in Fig. 2a. The situation is however different when both nonlinearities are present, leading to a unidirectional imaging produced either on transmission or on reflection with a properly chosen phase relations between the two types of the nonlinear response, as demonstrated in Fig. 2b. The nonlinear parameters of a VLSRR medium similar to the one used in the results shown in Fig. 1 but possessing two types of nonlinearity were used in the simulations.

ISBN 978-952-67611-2-1

We next consider similar processes of nonlinear image formation and nonlinear magneto-electric coupling in plasmonic nano-composites. Strong field enhancements created by the localized and propagating plasmon resonances lead to orders of magnitude increased effective nonlinear response, strongly enhancing the produced image in comparison with, e.g., a plain thin metal film. While a similar uni-directionality can be achieved by combining nanoelements acting as the effective electric and magnetic dipoles, an additional flexibility arises from utilizing the plasmonic nanoantenna properties of the constitutive elements, considering a non-reciprocity as nonlinear optical antennas pattern. An example of such a non-reciprocal nonlinear imaging obtained by the process of four-wave mixing in a system composed of nanowires coupled to a perforated film is presented in Fig. 3.

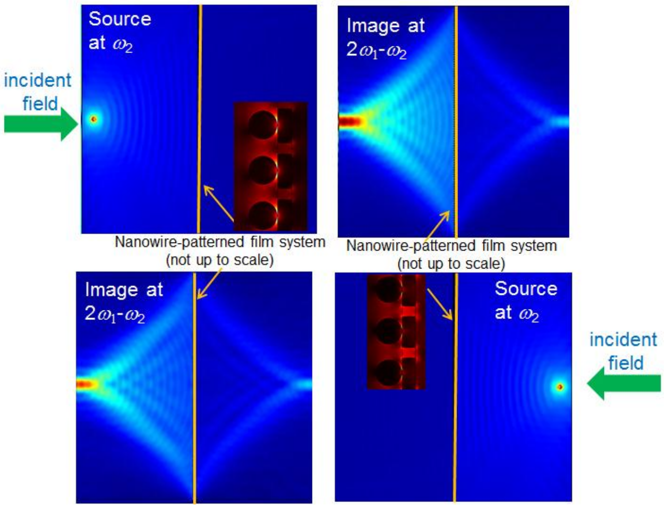

Fig. 3. An example of the non-reciprocal nonlinear imaging produced by the FWM process in the nanowire+perforated gold film. The point source at ω_2 and a plane-wave pump at ω_1 produce an image at $2\omega_1$-ω_2 located always on the left side from the system, independently on the source and pump direction.

References

[1] J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart. Magnetism from conductors and enhanced nonlinear phenomena. *IEEE Trans. Microwave Theory Tech.*, vol. 47, p. 2075, 1999.

[2] M. W. Klein, M. Wegener, N. Feth, and S. Linden, Experiments on second- and third- harmonic generation from magnetic metamaterials. *Opt. Express*, vol. 15, p. 5238, 2007.

[3] I. V. Shadrivov, S. K. Morrison, and Y. S. Kivshar. Tunable split-ring resonators for nonlinear negative-index metamaterials. *Opt. Express*, vol. 14, p. 9344, 2006.

[4] I. V. Shadrivov, A. B. Kozyrev, D. W. van der Weide, and Y. S. Kivshar. Tunable transmission and harmonic generation in nonlinear metamaterials. *Appl. Phys. Lett.*, vol. 93, p. 161903, 2008.

[5] V. M. Agranovich, Y. R. Shen, R. H. Baughman, and A. A. Zakhidov. Linear and nonlinear wave propagation in negative refraction metamaterials. *Phys. Rev. B*, vol. 69, p. 165112, 2004.

[6] N.M Litchinitser, I.R Gabitov, and A.I. Maimistov. Optical bistability in a nonlinear optical coupler with a negative index channel. *Phys. Rev. Lett*, vol. 99, p. 113902, 2007.

[7] E. Poutrina, D. Huang, and D. R. Smith, "Analysis of nonlinear metamaterials", *New J. Phys*., vol. 12, p. 093010, 2010.

[8] A. Rose, S. Larouche, D. Huang, E. Poutrina, and D. R. Smith, Nonlinear parameter retrieval from three- and four-wave mixing in metamaterials, *Phys. Rev. E* vol. 82, p. 036608, 2010.

[9] E. Poutrina, D. Huang, Y. Urzhumov, and D. R. Smith, Nonlinear oscillator metamaterial model: numerical and experimental verification, *Opt. Express*, vol. 19, p. 8312, 2011.

[10] D. Huang, E. Poutrina, and D. R. Smith, Analysis of the power dependent tuning of a varactor-loaded metamaterial at microwave frequencies, *Appl. Phys. Lett*., vol. 96, p. 104104, 2010.