Surface impedance and coupled-wave model for extraordinary optical transmission through realistic metallic screens

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Abstract

The paper proposes an analytical theory for extraordinary optical transmission (EOT) in metallic screens perforated by a periodic array of subwavelength holes. The theory is based on coupled-wave analysis and on the surface impedance concept. Full wave electromagnetic simulations are used to validate our model. The analytical model allows for a considerable reduction of CPU time in the analysis of EOT under normal or oblique incidence.

1. Introduction

Since EOT was first reported [1], a lot of effort has been dedicated to the characterization of the phenomenon and its potential applications in photonic circuits, optical sensing or fabrication of left-handed metamaterials. Surface plasmons (SPs) were soon the first explanation for EOT [2], but did not explain extraordinary transmission in metals at microwave frequencies [3] or in dielectric screens [4]. In recent works ([5], [6], [7]) we have developed analytical models for EOT from a waveguide theory perspective together with surface impedance approximation in order to characterize EOT in the whole frequency range where it has been reported. In this paper the analysis is extended to the case of oblique incidence in screens perforated by a periodic 2D array of holes. The results of the model are validated by full wave electromagnetic simulations.

2. Theory

The geometry of the unit cell of the structure is depicted in Fig. 1. We will consider oblique incidence of TE and TM waves with tangential E or H fields polarized along the main axes of the structure, as shown in Fig. 1(b) and Fig. 1(c).

The transversal components of the electromagnetic fields at both sides of the screen (regions 1 and 3) and inside the hole (region 2) can be expanded in terms of Bloch and waveguide modes and evaluated in the input and output surfaces of the screen. Specific boundary conditions are applied along the metallic screen and hole. Fields in regions 1 and 3 are related through a surface impedance matrix in the area of the screen

$$\begin{bmatrix} \boldsymbol{E}_{\parallel}^{(\mathbf{1})}(z=-t/2^{-}) \\ \boldsymbol{E}_{\parallel}^{(\mathbf{3})}(z=t/2^{+}) \end{bmatrix} \approx \overline{\overline{\boldsymbol{Z}}} \begin{bmatrix} \hat{\boldsymbol{z}} \times \boldsymbol{H}_{\parallel}^{(\mathbf{1})}(z=-t/2^{-}) \\ \hat{\boldsymbol{z}} \times \boldsymbol{H}_{\parallel}^{(\mathbf{3})}(z=t/2^{+}) \end{bmatrix} \quad \text{for } b/2<|x|,|y|<a/2, \tag{1}$$

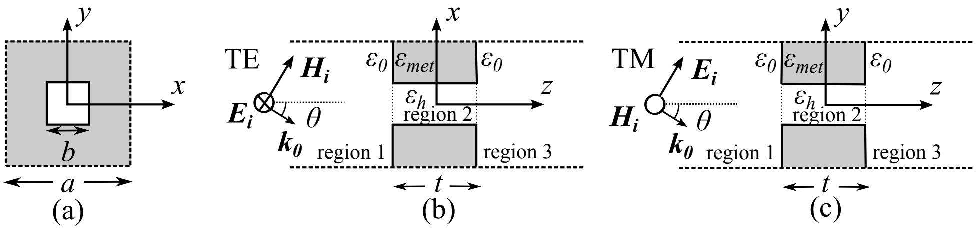

Fig. 1: Front (a) and side views (b, c) of the unit cell of the structure with the three regions in which the fields are expanded. The incident waves in the cases of TE and TM polarization are shown in (b) and (c) respectively. Due to the polarization of the incident waves and to periodicity, in (b) there are virtual electric walls in $y = 0, \pm a/2$ and in (c) there are virtual magnetic walls in $x = 0, \pm a/2$. The rest of lateral boundaries are periodic boundaries.

where $\boldsymbol{E}_{\|}^{(1)}$, $\boldsymbol{E}_{\|}^{(3)}$, $\boldsymbol{H}_{\|}^{(1)}$ and $\boldsymbol{H}_{\|}^{(3)}$ are vectors containing the transversal components of the fields in regions 1 and 3, and $\overline{\overline{\boldsymbol{Z}}}$ is the surface impedance matrix corresponding to a plain slab (without holes) for a given incident plane wave (see e.g. [8]). When applying (1) we are thus neglecting the perturbation associated to the presence of the holes. We are also assuming that the dominant refracted modes will also be connected by approximately the same surface impedance matrix so that (1) is satisfied by the total fields. This last approximation is valid for most metals as long as the transverse wavenumbers of the different modes are much smaller than the longitudinal wavenumber inside the metal [6].

In the area of the holes, continuity of transverse electromagnetic fields is imposed,

$$\begin{bmatrix} \boldsymbol{E}_{\|}^{(1)}(z=-t/2^-) \\ \boldsymbol{H}_{\|}^{(1)}(z=-t/2^-) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_{\|}^{(2)}(z=-t/2^+) \\ \boldsymbol{H}_{\|}^{(2)}(z=-t/2^+) \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{E}_{\|}^{(2)}(z=t/2^-) \\ \boldsymbol{H}_{\|}^{(2)}(z=t/2^-) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_{\|}^{(3)}(z=t/2^+) \\ \boldsymbol{H}_{\|}^{(3)}(z=t/2^+) \end{bmatrix}$$
$$\text{for } |x|, |y| < b/2. \qquad (2)$$

Following a procedure already employed in [7] in the case of slits configurations, boundary conditions (1) and (2) can be combined and rearranged in such a way that a sparse determinate system of equations is obtained after applying integral boundary conditions. Resulting system of equations yields the values of all the modal coefficients including the transmission coefficient.

3. Results

In Fig. 2 the transmission coefficients obtained with the reported model are compared with electromagnetic simulations using *CST Microwave Studio*. In order to obtain convergent results, the resolution of the higher modes inside the holes and in the input and output regions must be similar. We employed 2 modes inside the holes and $2(b/a) = 8$ modes in regions 1 and 3, with CPU times of ~ 0.5s vs CPU times of ~ 4min. per frequency value with the electromagnetic solver.

In Figs. 2(a) and 2(b) the metallic screen is modeled by a finite conductivity $\sigma = 59.6 \times 10^6$ S/m (corresponding to copper) with periodicity $a = 300\mu$m whereas in Figs. 2(c) and 2(d) the metal is modeled by a Drude Model for silver as in [7]. In such model, the frequency of collision of electrons was corrected in order to account for effects of thin thickness of the slab. The size of the holes was increased in the numerical computations by an effective depth $b_{\text{eff}} = b + 2\delta$, where δ is the skin depth of the metal, in order to model effects of field penetration in the inner walls of the holes. In the figure, frequencies are normalized to Wood's frequency which depends on the angle of incidence ($f_w(\theta) = c/[a\cos(\theta)]$ in case of an incident TE wave and $f_w(\theta) = c/[a(1 + \sin(\theta))]$ in case of an incident TM wave).

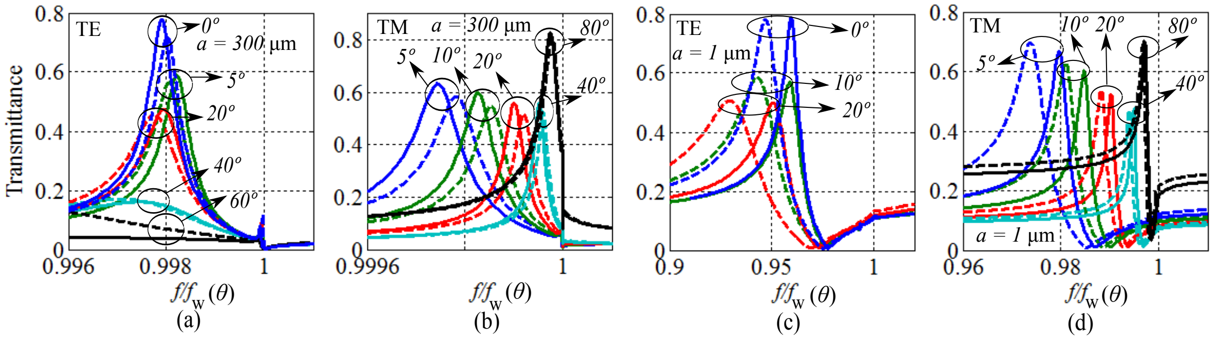

Fig. 2: Transmission through an array of square holes in a copper screen ($\sigma = 59.6 \times 10^6 S/m$) with periodicity $a = 300\mu$m (a) and (b) and in a silver screen modeled by a Drude permittivity with periodicity $a = 1\mu$m (c) and (d), holes size and thickness of the screens are $b = a/4$ and $t = a/20$ in all cases. Continuous lines correspond to mode matching model and dashed lines to CST simulations. In (a) incident wave is TE and Wood's frequencies range from 1.00 THz for $\theta = 0^o$ to 7.46 THz for $\theta = 60^o$. In (b) incident wave is TM and Wood's frequency range from 0.92 THz for $\theta = 5^o$ to 0.50 THz for $\theta = 80^o$. In (c) incident wave is TE and Wood's frequency range from 299.79 THz for $\theta = 0^o$ to 318.98 THz for $\theta = 20^o$. In (d) incident wave is TM and Wood's frequency range from 275.51 THz for $\theta = 5^o$ to 151.10 THz for $\theta = 80^o$.

4. Conclusion

An analytical model for the analysis of EOT through realistic metallic screens has been provided. The model, based on the surface impedance concept and the wave-coupling method, allows for a fast characterization of EOT at normal or oblique incidence. In future work, we will aim to extend the reported model to the characterization of fishnet metamaterials.

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