On the feasibility of pentamode mechanical metamaterials

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Abstract

Following the theoretical suggestion by Milton and Cherkaev in 1995, we fabricate pentamode metamaterials by dip-in direct-laser-writing optical lithography. Using finite element calculations and geometrical parameters corresponding to our fabricated three-dimensional microstructures, we find that the figure of merit, *i.e.*, the ratio of bulk modulus to shear modulus, can realistically be made as large as about 1,000. This result opens new horizons for transformation acoustics.

1. Introduction

It is interesting to translate transformation optics [1,2] to other types of waves such as acoustic waves. However, the three-dimensional elastodynamic equations are not invariant under coordinate transformations for scalar mass density and normal elastic materials [3]. In two dimensions or in thin plates, usual anisotropic elastic materials can suffice [4,5,6]. In three dimensions, one either needs materials with anisotropic mass density tensors [3,7,8,9] or pentamode materials [3,10,11,12] to implement the counterpart of invisibility cloaks or other devices. In 1995, Milton and Cherkaev [13] showed that all conceivable mechanical materials can be synthesized on the basis of pentamodes. Pentamodes are special in the sense that they avoid the coupling of compression and shear waves by making the bulk modulus, B, extremely large compared to the shear modulus, G, ideally infinitely large [13,14].

The pentamode ideal suggested by Milton and Cherkaev [13] is composed of double-cones arranged into a diamond crystal, the mechanical metamaterial. The tips of the cones touch each other only at singular points (see Fig. 2, however for d=0). To stabilize these structures, finite values of d are necessary that, however, will lead to a finite shear modulus. The question arises whether a satisfactory and feasible trade-off exists. Fortunately, we find that the answer is yes [15].

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2. Experiment

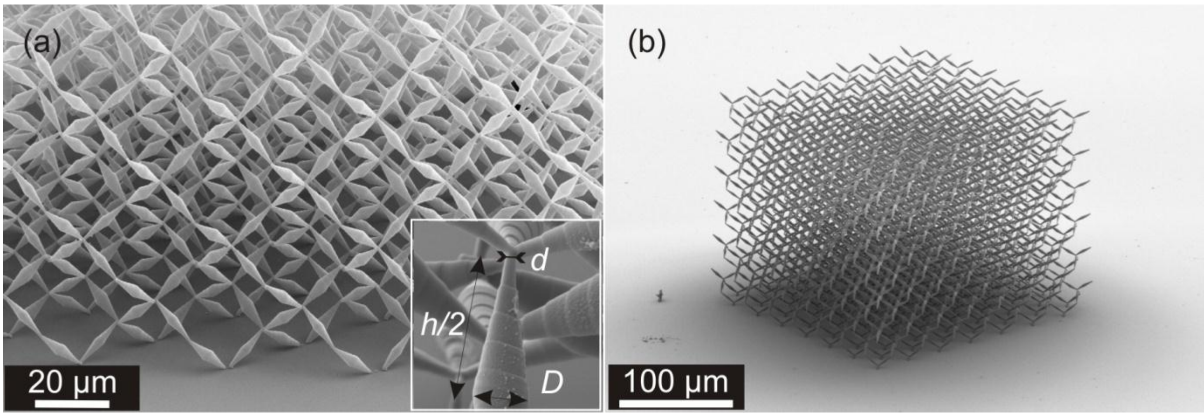

Fig.1: (a) Oblique-view electron micrograph of a polymer pentamode mechanical metamaterial fabricated by dip-in three-dimensional direct-laser-writing (DLW) optical lithography on a glass substrate with $h = 16.15$ μm and $D = 3$ μm. Importantly, the inset shows that the smallest accessible diameter of the connection regions of adjacent cones is about $d = 0.55$ μm. (b) Oblique-view electron micrograph of another polymer pentamode mechanical metamaterial with $7 \times 7 \times 6$ face-centered-cubic unit cells (hence total size 261 μm $\times$ 261 μm $\times$ 224 μm) and with $h = 16.15$ μm, $D = 3$ μm, and $d = 1$ μm. After Ref. [15].

Figs. 1(a) and 1(b) show polymer structures which we have fabricated using dip-in direct-laser-writing (DLW) optical lithography [16]. In brief, in contrast to regular DLW, we use the photoresist as the immersion fluid directly on the microscope objective lens. Fig. 1(a) exhibits an small diameter of only about $d = 0.55$ μm. Fig. 1(b) shows a larger view demonstrating the overall diamond-symmetry metamaterial.

2. Numerical simulations

As we will show next, the calculated ratio of bulk to shear modulus, the pentamode figure of merit (FOM), is already larger than 10^3 for the structure depicted in Fig. 1(a). Our corresponding numerical continuum-mechanics calculations use the commercial software package COMSOL Multiphysics (MUMPS solver) and typical polymer parameters, *i.e.*, a Young's modulus of 3 GPa and a Poisson's ratio of 0.4. The choice of these parameters is, however, not sensitive at all [15]. The pentamode bulk modulus, B, is directly obtained from its definition, *i.e.*, from applying a hydrostatic pressure and observing the resulting compression within the linear regime (*i.e.*, strains smaller than 1%). The shear modulus, G, is obtained by applying a shear force and calculating the displacement.

An overview on the resulting dependence of the B/G ratio = FOM on the small diameter, d, and the large diameter, D, is shown in Fig. 2 (complete curves are given in [15]). The corresponding primitive unit cells are shown for illustration. As expected, the B/G ratio increases with decreasing diameter d. It actually diverges for $d \to 0$ (not depicted). In contrast, the influence of D on the FOM is much less dramatic. We can summarize all of our numerical findings for pentamode structures like shown in Fig. 2 (*i.e.*, for $d < D \ll h$) by the simple approximate heuristic formula

$$\mathrm{FOM} = \frac{B}{G} \approx 0.63 \left(\frac{h}{d}\right)^2 \sqrt{\frac{h}{D}}.$$

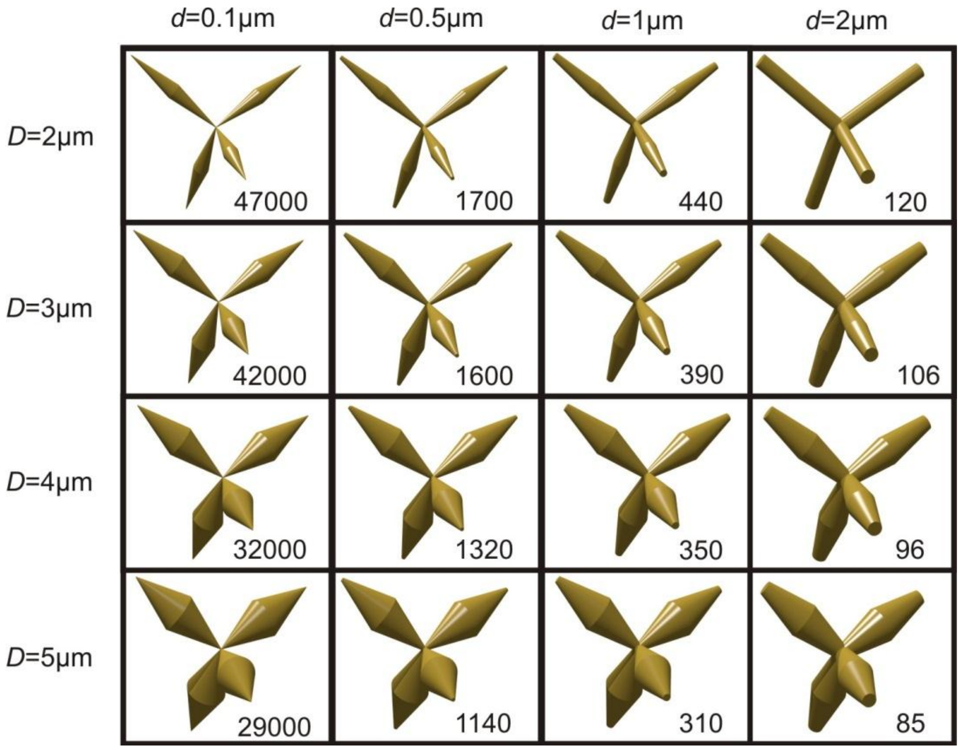

Fig.2: Ratio of bulk to shear modulus, the dimensionless pentamode figure of merit FOM, versus the small and the large diameter of the constituent cones, d and D. The overall size of the primitive cell is here fixed and directly connected to the length of the double-cones of $h = 16.15$ μm. The corresponding pentamode motifs are shown for illustration for each parameter combination. After Ref. [15].

4. Conclusion

Using state-of-the-art three-dimensional dip-in optical laser lithography [16], the pentamode metamaterial ideal can be approached [15]. Ratios of bulk to shear modulus in the range of 10^3 are realistically accessible [15], potentially opening new possibilities for transformation acoustics.

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