

# Transition to negative refractive index in plasma metamaterial with negative permeability

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#### Abstract

Metamaterials partly composed of discharge plasmas exhibit a number of distinguishable points from ordinary metamaterials. In particular, states with negative refractive index are triggered in a process with saddle-node bifurcations, and negative permeability is a key factor to achieve such transitions. In this paper, theoretical predictions and experimental verifications are demonstrated, which confirm that "plasma metamaterials" are quite nonlinear in the microwave range.

### **1. Introduction**

Metamaterials in the microwave range, which have exhibited a number of unique scientific features such as super lens and cloaking, [1,2] have been main research targets for wave media which cannot be achieved by ordinary materials in this decade. Recently, several methods to make them dynamic have been proposed to secure its flexible outputs for various objects. In this aspect, to obtain rapid responses as well as elaborate control of parameter values of metamaterials, we proposed discharge plasmas that play crucial roles in metamaterials, and we refer to them as "plasma metamaterials." [3] So far, we verified several schemes of plasma metamaterials in experiments, including dynamic negative-refractive-index (N) materials, [4,5] tunable supporters for spoof surface plasmon polaritons, [6] and chain-like structures of localized-surface-plasmon-like modes [7,8] as well as simple plasma photonic crystals.[7,9,10]

Permittivity of a bulk plasma  $\varepsilon_p$  is in the Drude model since free electrons determine its dielectric property, given as

$$\varepsilon_{\rm p} = 1 - \frac{\omega_{\rm pe}^2}{\omega(\omega + j\nu_{\rm m})},\tag{1}$$

where  $\omega/2\pi$  is the wave frequency, and  $v_{\rm m}$  is the electron elastic collision frequency against neutral particles in the gas phase.  $\omega_{\rm pe}$  is the electron plasma frequency, and  $\omega_{\rm pe}^2$  is proportional to electron density  $n_{\rm e}$ . When  $v_{\rm m}$  is negligible, that is, when the gas pressure is not so high with moderate electron temperature  $T_{\rm e}$ ,  $\varepsilon_{\rm p}$  is an almost purely real value, and it will become negative if  $\omega < \omega_{\rm pe}$  with fairly high  $n_{\rm e}$ . Simultaneous negative permeability  $\mu$  will lead to negative N.

In this report, we assume that a plasma in a negative  $\mu$  space is generated by a propagating microwave itself. In such a configuration,  $\varepsilon_p$  includes a nonlinear part because  $\omega_{pe}$  and  $n_e$  are as functions of the electric field of the microwave. The following sections describe transitions to a negative N state which is theoretically predicted and the values of  $\varepsilon_p$ ,  $\mu$ , and N observed in experiments.



# 2. Theoretical prediction and experimental results

In Ref. [11], we theoretically predicted transitions to negative N via quite nonlinear  $\varepsilon_p$ , which is briefly reviewed here. In a negative  $\mu$  space, microwaves at 2.45 GHz cannot propagate when  $\varepsilon_p$  is positive, and they can propagate when  $\varepsilon_p < 0$ ; the tendency is like a step-wise feature. Simultaneously,  $\varepsilon_p$  includes a nonlinear part with an exponential function due to an ionization coefficient, and the synthesis of these two effects lead to saddle node bifurcations of  $\varepsilon_p$  at points A and B as a function of the microwave electric field, as shown in Fig. 1. This indicates that high- $n_e$  plasmas with negative  $\varepsilon_p$  are generated, and low- $n_e$  plasmas with positive  $\varepsilon_p$  are not generated or exist only in a transient state.



Fig. 1: Bifurcation diagram with phase portrait.[11] Microwave propagation is numerically calculated at 2.45 GHz, and  $n_e$  is derived via ionization coefficient and particle balance.

In experiments, we installed a negative- $\mu$  metamaterial in a rectangular waveguide at 2.45 GHz, immersed in a vacuum chamber at 100 Pa of Ar. The negative- $\mu$  metamaterial composed of conventional double split ring resonators [12] was about 10 cm long, and  $\mu$  was estimated to be -0.54+0.13j at 2.45 GHz by a parameter retrieval method. Then, we launched microwaves up to 500 W with orientation of magnetic fields for matching of resonance with the negative- $\mu$  metamaterial. When the microwave power reached at 410 W,  $\varepsilon_p$  became -2.2+0.1j with  $n_e = 2.4 \times 10^{11}$  cm<sup>-3</sup> and  $T_e = 1-3$  eV, which were measured by a conventional single probe method developed for plasma diagnostics. When we decreased the microwave power down to 80 W, we observed twinkling discharges, but we could not observe positive  $\varepsilon_p$  when the plasmas were generated. This fact coincides with the theoretical prediction shown above.

# 3. Conclusion

We demonstrated transitions of N in a plasma metamaterial with negative  $\mu$  generated by propagating microwaves. When plasmas are not generated, N is imaginary; when the plasmas are generated, N jumps to a real and negative value (-1.2 as a maximum observed value). Such features were predicted in the theoretical approach, and verified by experimental results.



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