Nonlinear magnetoelastic metamaterial using gravitational restoring force

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Abstract

In this paper we present a novel design and preliminary demonstration of a nonlinear magnetoelastic metamaterial. The structure consists of carefully engineered split rings with mechanical counterbalance allowing gravity to serve as the elastic restoring force countering the attraction between the currents induced on the split ring resonators suspended with mechanical degrees of freedom in the waveguide. Measurements show shift in resonant frequency of the structure due to changed mutual orientation of the rings in response to increased incident power.

1. Introduction

Structural tuning of metamaterials has been investigated in recent years. Changing mutual position of metamaterial elements results in the resonance frequency shift because of changed coupling between the elements [1]. Tuning by lateral displacement of metamaterial layers with respect to each other [2] as well as relative rotation of the layers containing resonators [3] has been demonstrated. More complex ‘pop up’ structures rotating individual elements showed appearance of previously absent types of resonant responses [4]. However in all these structures free movement of the resonators was restricted.

Other types of metamaterials being actively researched were nonlinear metamaterials [5] where tuning was achieved by changing the power level of the incident field itself. The nonlinearity was introduced either by adding a nonlinear element like varactor diode or by embedding into a nonlinear medium.

Recently a novel concept of magnetoelastic metamaterials was introduced [6]. The principal difference was that the resonators were allowed to move freely in response to the incident electromagnetic radiation thus combining the approaches mentioned above. The varying incident magnetic field induced circular currents on the rings. Once the magnitude of the incident radiation was large enough the rings would attract to each other. The rings were assumed to be embedded in the elastic medium that provided the restoring force preventing the rings from collapsing onto each other. Any displacement of the resonators would result in change of the resonant frequency of the system. The system showed complex nonlinear and bistable behaviours without the introduction of traditional nonlinear materials.

We propose an engineered version of this type of metamaterial with integrated suspension mechanism that can be easily fabricated in bulk quantities. Using standard photolithography fabrication and micromilling techniques high precision and low parameter variation across multiple elements can be achieved. Also the method allows utilising a wide range of more complex split ring resonator shapes. The restoring force in our case is the force of gravity only. We theoretically investigate the magnitudes and directions of the forces involved in the movement of the resonators.

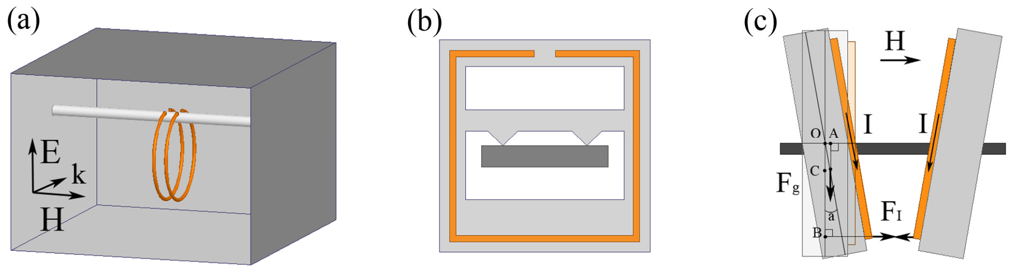

Fig. 1: (a) Initial split ring resonators in the waveguide (b) proposed split ring resonator design: ring side size a = 9 mm, trace width *wr* = 0.2 mm, substrate size *b* = 10 mm, ring width *ws* = 1 mm, thickness *t* = 0.1 mm (c) schematic side view showing the forces acting on two resonators suspended at *d* = 1 mm apart from each other.

2. Model Design

Fig. 1(a) shows the original design [6], where copper wires were bent to form split ring resonators. It is difficult to produce a number of resonators with the same geometrical parameters using this method. We propose the design of metamaterial with integrated suspension mechanism fabricated by standard photolithography fabrication process. Fig. 1(b) shows the proposed split ring resonator design. The substrate chosen for the design is copper clad Rogers Ultralam material with dielectric constant $\varepsilon = 2.9$ and loss tangent $\delta = 0.0025$ available in thicknesses of 0.1 mm and less. Its density of 1.4 g/cm^3 is lower than other available circuit materials and allows reducing the mass of the resonator.

Horizontal strip of substrate in the middle serves as a suspension mechanism. In order to reduce the friction, area of contact with the support was minimised. The teflon based support has grooves defined along the plane of the resonators preventing them from sliding towards each other. The unwanted area of the substrate can be removed using either laser or mechanical milling. By varying the position of the suspension as well as the amount of substrate around the bottom side of the ring the position of the centre of gravity of the ring and correspondingly the amount of the restoring force can be controlled.

Fig. 1(c) shows the diagram of the forces acting on the rings. The rings are suspended in the waveguide in close proximity *d* to each other. The metal coated sides face each other and the gaps are located at the top of the rings. At the resonant frequency the incident magnetic field passing through the rings induces circular currents that attract each other with Ampere force F_I. The maximum of the current distribution is along the bottom side of the rings. Since the translational movement of the rings is restricted by the grooves in the support, each ring rotates towards the other around the corresponding suspension axis O passing through the tips of the suspension mechanism. The response time of this mechanical movement is much larger than the period of incident wave oscillations.

The friction and air resistance forces oppose the initial movement of the resonators and thus present a power threshold for nonlinear action. In this study, we assume that their magnitudes were sufficiently small to be neglected. The main restoring force is the gravitational force that rotates the rings in the opposite direction in order to return them to the initial state with the lowest position of the centre of gravity C. The moments of the Ampere force and the gravitational restoring force around the axis of rotation O are given by $M_I = l_I F_I$, $M_g = l_g F_g$ where l_I = OB and l_g = OA are the corresponding moment arms. The forces can be calculated using the following formulae [6, 7]:

$$F_g = mg\,, \qquad F_I = \frac{\mu_0 I_1 I_2}{2\sqrt{4+b^2}}\left(\varepsilon\frac{2+b^2}{b^2}-\kappa\right),$$

where *m* is the mass of the suspended structure including the split ring and the substrate, *g* is gravitational constant, I_1 and I_2 are currents induced on the rings, ε and κ are elliptic integrals with the parameter $\varkappa^2 = 4/(4+b^2)$, where $b = d/(a/2)$. As the ring rotates the moment arm of the Ampere force l_I reduces and the moment arm of the gravitational force l_g increases. The equilibrium is achieved when $M_I = M_g$. In order to reduce the minimum force required to move the resonators their centre of gravity C needs to be close to the axis of rotation O. So the gravitational force moment arm l_g is reduced and the lever effect is magnified. This should reduce the amount of incident power needed to induce sufficiently strong currents for the rings to move. That is why it is beneficial to have the axis of rotation in the middle rather than at the top of the structure. However one must ensure that the centre of gravity of the resonators is below the axis of rotation, otherwise stable suspension can not be achieved.

3. Results and Discussion

As a preliminary test of the functionality of the structure, we placed an identical pair of rings within a WR187 rectangular waveguide and applied low (-5dBm) and high (28dBm) incident power levels. Fig. 2 shows measured transmission through the structure. At high incident power the rings swing towards each other by rotating around the corresponding suspension axis. This shift in mutual positions of the resonators results in resonant frequency shift of the structure by around 20 MHz. To ensure that the shift is indeed coursed by change in mutual position of the rings the same test was performed on single ring and no frequency shift was observed when switching to higher power level.

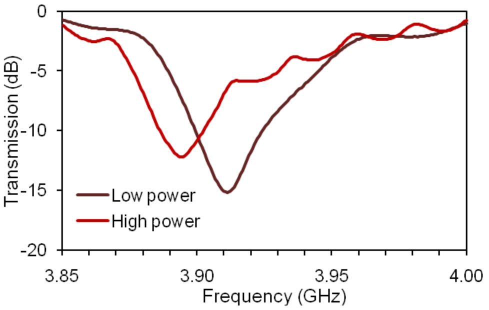

Fig. 2: Transmission characteristics of the structure measured at low and high input power levels.

4. Conclusion

In this paper we have proposed and presented preliminary demonstration of a magnetoelastic metamaterial design suitable for fabrication by standard photolithography technique. The forces involved in the movement of the resonators suspended with the mechanical degrees of freedom were theoretically investigated. Methods for optimising the design in order to minimise the gravitational restoring force were proposed. Measurements demonstrated the shift of the resonant frequency of the structure in response to increase of incident radiation power. The structure we have introduced will be suitable for forming arrays of resonators allowing construction of nonlinear volumes of material. Further, the ability to engineer the linear and nonlinear behaviour will enable exploration of materials with graded characteristics, opening significant opportunities for exploration.

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