# Dispersive TLM Model of Lossy GRIN MTM Structures

**T. Asenov**[1], **N. Doncov**[1], **B. Milovanovic**[1]

[1]Department of Telecommunications, Faculty of Electronic Engineering, University of Nis
Aleksandra Medvedeva 14, 18000 Nis, Serbia
Fax: + 381–18588399; email: tatjana.asenov@elfak.ni.ac.rs

## Abstract

In this paper, dispersive numerical model of lossy composite right-left handed (CRLH) structures with gradient refractive index profiles is considered. Model to account for dispersive properties of left-handed metamaterials in the time-domain, is implemented into 3-D Transmission-Line Matrix Z-transform method. The accuracy, efficiency and stability of proposed model are verified for several gradient refractive index profiles across RH-LH interface using analytical solutions.

## 1. Introduction

Unique properties of left-handed metamaterials (LH MTM) have been exploited for the realisation of numerous advanced microwave components [1]. In recent years, CRLH structures with gradient refractive index profiles (so-called GRIN MTM) exhibiting the phenomenon of electromagnetic (EM) enhancement at the boundary surface, low return loss and a significant decrease in the effect of geometric aberration have been intensively studied. Manipulation of EM radiation in MTM uncommon for natural materials and transformation optics represent the fundamentals of GRIN structure applications. The simultaneous gradual change of the permittivity and permeability gives additional degree of freedom in composite GRIN MTM structures design. Another advantage is the effortless impedance matching in free space and improved performances at microwave and optical frequencies. Thus, a number of GRIN applications have been proposed [1].

Realisation of GRIN structures could be challenging at high frequencies since standard fabrication methods are feasible for planar structures with a limited number of layers. Only moderate refractive index changes can be achieved. Hence, the effects predicted by analytical solutions derived for some profiles are limited [2-3]. Therefore, numerical characterisation of GRIN MTM is very important since it enables the investigation of profiles possible to manufacture as well as possibly useful profiles without derived analytical solution. Incorporated LH MTM models into differential numerical techniques in the time-domain are of special interest as they allow the time-harmonic and transient MTM simulation and dispersive behaviour analysis. A model based on Drude function, capturing frequency-dependent properties of LH MTM, and bilinear transformation of this dependence in the discrete time-domain is presented in [4] for the TLM method based on Z-transform [5]. In this paper, we extend this dispersive model, originally incorporated into 1-D TLM mesh, to 3-D TLM mesh and apply it to describe lossy GRIN MTM structures with different gradient of real part of refractive index across the interface between CR and LH materials. The numerical simulation of EM wave interaction with differently graded lossy interface reflects close agreement with the analytic solutions and model accuracy and efficiency for GRIN MTM modelling.

## 2. Dispersive 3D TLM Z-transform Model of LH MTM

TLM Z-transform model has been presented and discussed in detail in [4]. It is developed by using the Drude dispersion model for electric and magnetic susceptibilities in the s-domain:

$$\chi_e(s)=\chi_{e\infty}+\frac{\omega_{pe}^2}{\gamma_e}\left(\frac{1}{s}-\frac{1}{s+\gamma_e}\right),\ \chi_m(s)=\chi_{m\infty}+\frac{\omega_{pm}^2}{\gamma_m}\left(\frac{1}{s}-\frac{1}{s+\gamma_m}\right) \tag{1}$$

but it can be easily modified if LH MTM properties are given by electric and magnetic conductivities:

$$\sigma_e(s)=\frac{\sigma_{e0}}{1+s\tau_e},\ \sigma_m(s)=\frac{\sigma_{m0}}{1+s\tau_m} \tag{2}$$

In Eqs.(1-2), $\omega_{pe,m}$, $\gamma_{e,m}$ and $\sigma_{e,m0}$ are electric or magnetic plasma frequencies and corresponding collision frequencies and static conductivities, respectively. Electric and magnetic collision times can be expressed through corresponding collision frequencies as $\tau_{e,m}=1/\gamma_{e,m}$. For a LH MTM matched to free space, the static electric and magnetic conductivities are related by $\sigma_{m0}=(\eta_0)^2\sigma_{e0}$. $\eta_0$ is the wave impedance of free space. Also, this approach can be easily modified for Lorentzian or higher-order material responses. The algorithm of TLM Z-transform approach, in which dispersive model is incorporated is given in Fig.1 [5]. The Drude representation of frequency-dependant EM properties of LH MTM and bilinear Z-transformation of this dependence into the time-domain are performed in the block $\underline{\underline{t}}(z)$. Fig.2 illustrates the calculation of the y-component of the electric field for the 3-D TLM modelling, performed in $\underline{\underline{t}}(z)$. Other electric/magnetic field components can be similarly calculated.

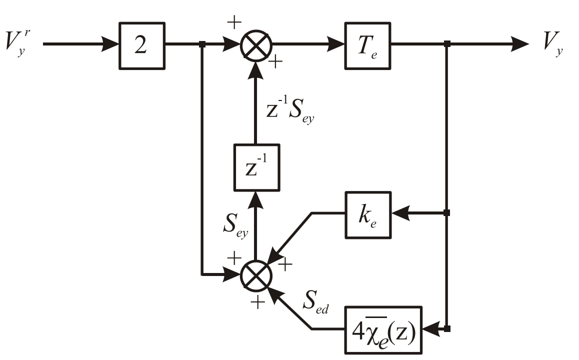

Fig.2: Dispersive 3D TLM Z-transform model for LH MTM - calculation of $E_y$ in the time-domain

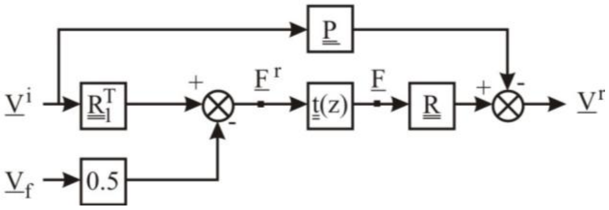

Fig.1: Signal flow diagram of the general algorithm of TLM method based on Z-transform

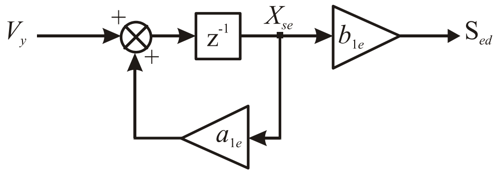

Fig.3: Calculation of accumulator $S_{ed}$

The accumulator $S_{ed}$ in the block $4\overline{\chi_e}(z)$ describes the frequency dependence of LH MTM EM properties using their values in previous time-steps (Fig.3). In addition, the following modifications are required in the expressions in [4] in order to incorporate dispersive model into 3-D TLM mesh:

$$b_{1e}/4=K_e 4\gamma_e\Delta t/[2B_e^3],\ T_e=(4+g_e+4\chi_{e0})^{-1},\ k_e=-(4+g_e-4\chi_{e1}) \tag{3}$$

## 3. Numerical results

Dispersive 3D TLM Z-transform model is used to investigate EM wave propagation across an interface between CR and LH materials with an abrupt refractive index profile (Fig.4, line 1) and symmetric gradient of refractive index. This gradient is taken to be either a tanh or cos function (lines 2 and 3 in Fig.4). In all cases there is an analytical solution for arbitrary losses and it is valid for entirely arbitrary choice of the EM properties frequency-dependence [2,3]. The lossy interface is illuminated by a 300THz plane wave (i.e. λ=1 μm). It is assumed that imaginary part of relative permittivity and permeability is 0.02 at 300THz. The electric field distribution observed near the lossy interface for considered refractive index profiles is shown in Figs.5, 6 and 7. A close agreement between the analytical and 3D TLM Z-transform model results is demonstrated.

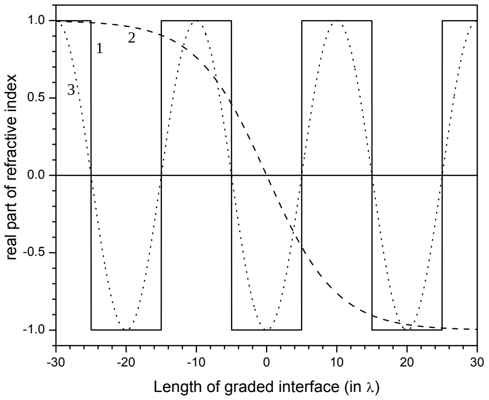

Fig.4: Variation of real part of refractive index across the lossy RLH interface for different profiles

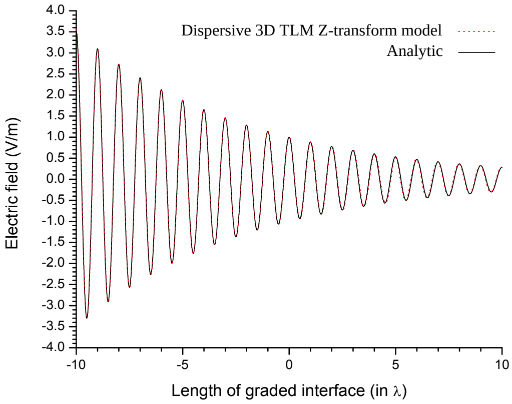

Fig.5: Comparison of the analytical and numerical results for the lossy RLH interface with abrupt profile

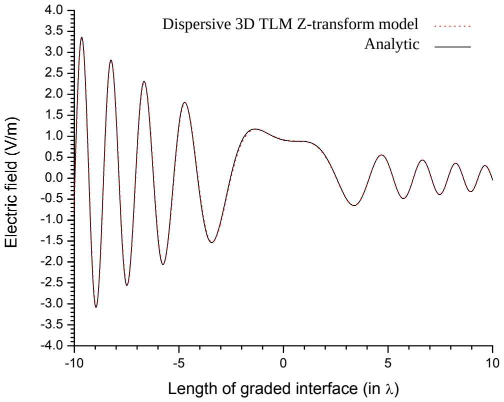

Fig.6: Comparison of the analytical and numerical results for the lossy RLH interface with tanh profile

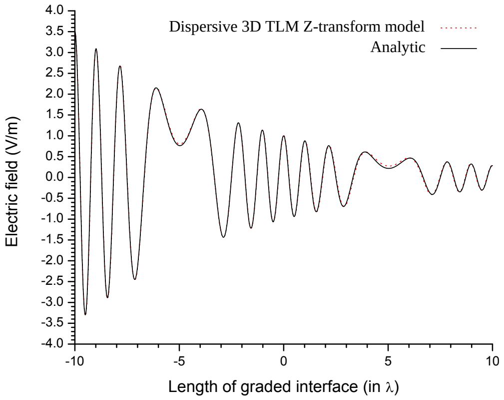

Fig.7: Comparison of the analytical and numerical results for the lossy RLH interface with cos profile

## 4. Conclusion

In this paper, the enhanced Z-transform based TLM method for direct time-domain MTM modelling is applied to GRIN MTM with different refractive index profiles. The numerical results confirm theoretically predicted behaviour of EM wave interaction with lossy graded interface and illustrate the stability, accuracy and applicability of proposed approach for entirely arbitrary refractive profiles.

## Acknowledgment

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