Non-linear-transformation based cylindrical cloaks and their practical advantages

V. S. Asadchy[1,2], I. A. Faniayeu[1], I.V. Semchenko[1], S.A. Khakhomov[1]

[1]Department of Physics, Gomel State University
Sovyetskaya Str. 104, 246019, Gomel, Belarus
Fax: + 375–232578111; email: picassonok@gmail.com
[2]Department of Radio Science and Engineering, Aalto University
Espoo, Finland

Abstract

Constitutive electromagnetic parameters of cloaks based on non-linear (quadratic and exponential) coordinate transformations are considered in this paper. The paper includes analysis and comparison of such parameters and parameters of cloaks based on the linear transformation found previously by other researchers. The shortcomings and advantages of found parameters are presented in this work. Also the results of numerical analysis of the cloak parameters done with COMSOL Multiphysics are shown.

1. Introduction

In the last years prospects of creation electromagnetic cloaks has been attracting attention of scientists. There are many different methods of electromagnetic cloaking of objects. The wave flow method is one of them [1,2]. The necessary constitutive parameters (tensors of permittivity and permeability) of cylindrical cloak structures were obtained in [3-5]. The best for practical applications constitutive cloak parameters based on the linear transformation are presented in paper [3]. There are experimental results of cloaking of cylindrical objects in aforementioned paper. In this paper theoretical derivation of constitutive cloak parameters based on non-linear (quadratic and exponential) transformations is presented. Also this article includes its analysis and comparison with parameters [3] in terms of practical applications. In the paper it is shown that the proposed cloak parameters have some advantages. They offer opportunities of arbitrary stretching electromagnetic waves inside cloak and decreasing absorption of energy.

2. Determination of cloak parameters

Cylindrical shell of a cloak structure has the inner radius a and external radius b. In paper [1] the following linear coordinate transformation was proposed:

$$r'(r) = \frac{b-a}{b} \cdot r + a, \qquad (1)$$

where r and r' are radius vectors in the original and curved spaces, respectively. According to this equation the front of the incident wave is distorted uniformly. It is necessary to take in account some limitations:

$$r'(0) = a,\; r'(b) = b,\; \left.\frac{dr'}{dr}\right|_{r=0} > 0,\; \left.\frac{dr'}{dr}\right|_{r=b} > 0. \quad (2)$$

In present paper the following quadratic

$$r'(r) = pr^2 + qr + a, \quad p = \frac{b-a}{\gamma \cdot b^2},\; q = \frac{(b-a)(\gamma-1)}{\gamma \cdot b},\; |\gamma| > 1, \quad (3)$$

and exponential transformation

$$r'(r) = \frac{1}{\sinh(\alpha b)}[b\sinh(\alpha r) - a\sinh(\alpha r - \alpha b)] \quad 0 < |\alpha| < \frac{1}{b}\ln(\frac{b}{a} + \sqrt{\frac{b^2}{a^2} - 1}) \quad (4)$$

are investigated. Tensors of permittivity and permeability of cloak were obtained for quadratic transformation (3):

$$\varepsilon_{rr} = \mu_{rr} = \frac{r^2 + (B(r)-a)(B(r)-a+2r)}{A(r)\cdot r^2},\; \varepsilon_{\varphi\varphi} = \mu_{\varphi\varphi} = \varepsilon_{zz} = \mu_{zz} = \frac{1}{A(r)}, \quad (5)$$

where other tensor components are equal to zero, and the introduced functions *A* and *B* are calculated as following:

$$A(r) = \sqrt{q^2 - 4p(a-r)}, \quad B(r) = \frac{(A-q)^2}{4p}. \quad (6)$$

Tensors of permittivity and permeability of the cloak were obtained for exponential transformation (4):

$$\varepsilon_{rr} = \mu_{rr} = T(r) + \frac{R^2(r)\cdot\rho(r)\cdot(r\cdot T(r) + R(r))}{r^2\sinh(\alpha b)}, \quad \varepsilon_{\varphi\varphi} = \mu_{\varphi\varphi} = \varepsilon_{zz} = \mu_{zz} = T, \quad (7)$$

where the following functions were introduced:

$$T(r) = \frac{R(r)\cdot\sinh(\alpha b)}{r\cdot\sinh(\alpha b) + R^2(r)\cdot\rho(r)}, \quad (8)$$

$$\rho(r) = \frac{\alpha\cdot(b\cosh(\alpha r) - a\cosh(\alpha r - \alpha b))}{r} + \frac{a\sinh(\alpha r - \alpha b) - b\sinh(\alpha r)}{r^2}. \quad (9)$$

Function *R(r)* is inverse function of *r'(r)* and it can be obtained from (4).

3. Practical advantages of cloaks based on non-linear transformations

Arbitrary choice of constants α and γ in equations (3-4) allows us to design cloak structures with different properties. *Z*-component of electric field for different cloak parameters based on a quadratic transformation is shown in Fig. 1. The incident electric field is equal 1 V/m. The working frequency is 3 GHz. As one can see from Fig. 1, the use of a non-linear transformation makes it possible to create a cloak with arbitrary stretching of electromagnetic waves inside. Composite materials can be used for practical realization of required materials [3,6]. Concentration of metal inclusions in composite material can be determined using the Clausius-Mossotti relation. Some part of the incident wave energy is absorbed in consequence of frequency dispersion and loss in metal inclusions. The use of non-linear tranformations for cloaking makes it possible to decrease energy loss.

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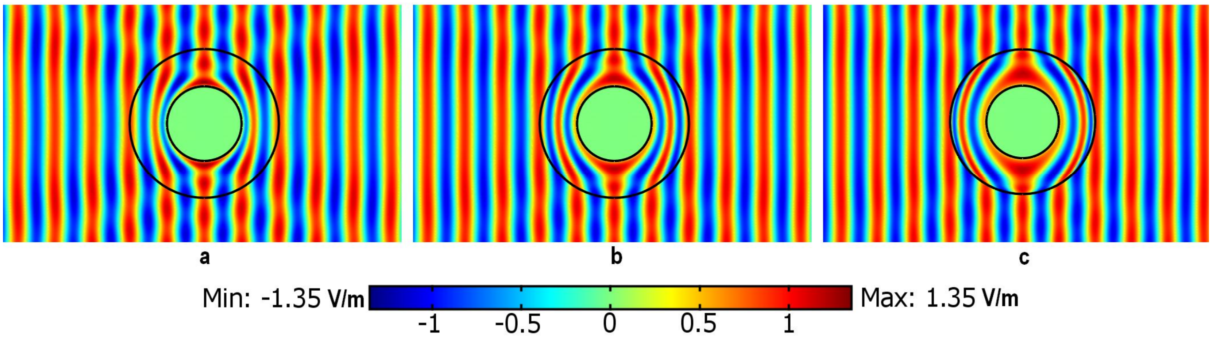

Fig. 1: E_z distribution for quadratic transformation based cloak. (a) γ=1,4. (b) γ=1000. (c) γ= -1,4.

The power loss in a cloak which consists from metallic inclusions can be determinated as following:

$$P_{abs} = \pi h \omega \varepsilon_0 \int_a^b r \cdot (\left|E_r^2\right| \varepsilon''_{rr} + \left|E_\varphi^2\right| \varepsilon''_{\varphi\varphi} + \left|E_z^2\right| \varepsilon''_{zz}) dr, \tag{10}$$

where h is the height of the cloaked cylinder (losses due to magnetic field components are given by a similar formula). This integral was calculated for cloak based on linear and non-linear transformations using the Clausius-Mossotti relation and numerical simulations. The losses in the exponential-transformation cloak are 22% smaller than in the linear-transformation cloak of the same size. This is a significant advantage of the constitutive parameters of cloaks based on non-linear transformations.

4. Conclusion

Described in this paper parameters of cloak structure are more preferable for practical realization than the constitutive parameters based on linear coordinate transformations. The cloak structure based on non-linear transformations absorbs less energy.

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