

# Phase Conjugation at Normal Incidence of Signal Wave on Active Metasurface with Linear and Nonlinear Huygens Sources

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#### Abstract

A possibility of almost perfect phase conjugation on electrically thin nonlinear sheet/"metasurface" with chiral metaparticles is shown for the normal incidence of the signal wave, using non-degenerate interaction with slightly different frequencies of the incident and phase conjugated waves. Under these conditions, the use of linear and nonlinear Huygens sources for both linear (signal) and nonlinear (phase conjugated) waves ensures zero or very small reflection for both of them.

## 1. Introduction

The process of wave front reversal also known as *phase conjugation* has been investigated for electromagnetic waves from microwaves to the optical wavelength range. Phase conjugation has been demonstrated in both isotropic and anisotropic (ferrite) media [1]. In [2] it was proposed to use phase conjugation of electromagnetic fields for creation of perfect lenses. Recently, linear Huygens sources have been used in the linear regime as effective metamaterial antennas, without (or with very small) reflections [3]. The new idea [4] is to use an active surface with nonlinear metaparticles, working as Huygens sources both in linear and nonlinear regimes. In perspective, this could lead to realization of reflectionless superlens based on three-wave interactions and other applications. Here we show a possibility to design a nonlinear active surface with metaparticles which constitute simultaneously linear and nonlinear Huygens sources. This will be shown for a particular example of the normally incident signal wave (see some discussion concerning arbitrary incidence angles in [5]). Three-wave interaction processes are considered as a basis of "reflectionless phase conjugation" on active surfaces, providing "perfect lensing". The relation  $\omega_p = \omega + \omega_{pc}$  is valid where  $\omega_p, \omega$  and  $\omega_{pc}$  are the frequencies of the pumping wave (or pumping voltage applied to all elements with quadratic nonlinearity, loading active metaparticles), the signal, and the phase conjugated waves, respectively. We show that the only feasible regime is the non-degenerated coupling, when  $\omega \neq \omega_{pc}$  (but  $\omega \approx \omega_{pc}$ ).



### 2. Formulation of the problem and conditions of reflectionless phase conjugation

Consider conditions, to which electric and magnetic surface polarization densities should satisfy, to provide phase conjugation for the plane wave with components  $(E_x, H_y)$  or  $(H_x, E_y)$ , which falls normally from  $z=-\infty$  on the plane z=0. If chiral particles are used on the active surface, both of the modes  $E_x, H_y$  and  $(H_x, E_y)$  would be excited, generally speaking, in reflected and transmitted fields. In spite of this, because corresponding surface polarizations  $\vec{P}^{(e)} = (P_x^{(e)}, 0, 0), \vec{P}^{(m)} = (0, P_y^{(m)}, 0)$  and

 $\vec{P}^{(e)} = (0, P_y^{(e)}, 0), \vec{P}^{(m)} = (P_x^{(m)}, 0, 0)$  are orthogonal to each other, we can consider the conditions, which surface polarizations should satisfy, to provide "reflectionless phase conjugation" separately for each of the two modes. Denoting the amplitudes of the incident, reflected and transmitted electric fields as  $A_{E_i}, A_{E_{ir}}, A_{E_{ir}}$ , respectively, the "ideal" ("reflectionless") phase conjugation is described by the relations

$$A_{Er} = 0 \tag{1 a}$$

$$A_{Et} = C_{pc} A_{Ei}^* \tag{1 b}$$

where  $C_{pc}$  is an arbitrary complex constant, and "\*" means complex conjugation. For nondegenerated coupling, amplitudes  $A_{Et}$ ,  $A_{Et}^*$  belong to the transmitted (phase conjugated) and incident (signal) waves having different frequencies. Consideration of an incidence, reflection and transmission with proper boundary conditions, yields the conditions of the absence of reflection, on a given frequency, in the form

$$P_{y}^{(m)} = cP_{x}^{(e)} \text{ or } P_{x}^{(m)} = -cP_{y}^{(e)}$$
(2)

for the incident mode  $(E_x, H_y)$  and surface polarization  $\vec{P}^{(e)} = (P_x^{(e)}, 0, 0)$ , and incident mode  $(H_x, E_y)$ and surface polarization  $\vec{P}^{(e)} = (0, P_y^{(e)}, 0)$ ,  $\vec{P}^{(m)} = (P_x^{(m)}, 0, 0)$ , respectively. In the case of chiral particles, both surface polarizations are excited even if only one of above mentioned electromagnetic mode incidents on the active surface. Note, that, in the case of non-degenerated coupling, "incident" phase conjugation wave is absent, while possible "transmitted" and "reflected" waves on this frequency are excited due and proportional to nonlinear surface polarization. The last, in turn, is excited due to mixing, on nonlinear elements (diodes/capacitance), embedded into metaparticles, of the field of incident wave and pumping  $u_p$ , applied to the nonlinear element (with quadratic nonlinearity). Therefore, under non-degenerated coupling, condition (1 b) is satisfied "automatically. To provide reflectionless phase conjugation, eq. (1 a) should be satisfied both for signal and phase conjugated waves. Respectively, first and/or second of eqs. (2) should be also satisfied for both of the frequencies  $\omega, \omega_{pe}$ . Whether first, second, or both conditions of (2) should be satisfied, depends on the polarization of the incident wave and on whether one use chiral metaparticles or not.

#### 3. Results of modelling

The "elementary cell" of used chiral metaparticles, namely,  $\Omega$ -particles, and corresponding equivalent circuit are shown in Fig. 1 (left) and Fig. 1 (right), respectively.



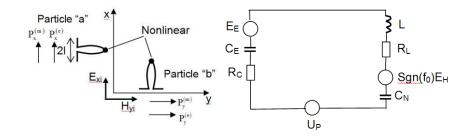


Fig. 1 Elementary cell of the metasurface with chiral particles (with magnetic loops laying in the plane xy) (left) and equivalent circuit describing metaparticle (right).  $P_{x,y}^{(e,m)}$ -surface polarizations, excited in chiral particles a and b in response to incident polarization ( $E_{xi}, E_{yi}$ );  $E_{E,H}$ ,  $C_E$ , L and  $R_{C,L}$  denote electromotive forces, connected with electric dipole and magnetic loop of the chiral particle, capacity, connected with the stem in chiral particle, inductance of the magnetic loop and effective resistances, describing losses in magnetic and electric dipoles, respectively;  $C_N, U_p$  describe nonlinear capacity (while quadratic nonlinearity is used) embedded into

chiral particle and amplitude of pumping voltage on the frequency  $\omega_p$ , respectively.

Analysis of the conditions (2) and equivalent circuit depicted in Fig. 1 (right) shows that it is possible to provide self -conjugation with zero reflection on signal frequency and small "reflection", of the order of  $|(\omega - \omega_{pc}) / \omega| \sim 0.1$ , on the frequency  $\omega_{pc}$ . Note that the "reflection" on frequency  $\omega_{pc}$  is counted respectively to the amplitude of transmitted wave, instead of incident amplitude, which is zero for the phase conjugated wave. Conditions of such a "reflectionless phase conjugation" are found in terms of the characteristics of chosen metaparticles, shown in Fig. 1 (left). Evaluations for  $\omega \sim 2\pi \cdot 14 \cdot 10^{10} s^{-1}$  show that effectiveness of power transformation to phase conjugated wave, normalized on signal power, can be of order of  $10^{-5}$ . More details will be published elsewhere.

#### 4. Conclusions

A possibility of almost perfect phase conjugation on active metasurface is shown for the normal incidence of the signal wave and non-degenerate three-wave interaction with slightly different frequencies of the incident signal and phase conjugated waves. The use of linear and nonlinear Huygens sources for both linear (signal) and nonlinear (phase conjugated) waves ensures zero or very small reflection for both of them. Relative power transformation for phase conjugated wave of order of  $10^{-5}$  is possible, what is quite enough at least for experimental demonstration.

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