Application of Optical Transformation for Wavefront Conversion by Transmission Line Method

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Abstract

Cylindrical to planar optical transformation is proposed using a two-dimensional transmission line metamaterial. The simplified and realizable constructive parameters of the engineered medium are presented based on the introduced optical transformation. The design of transmission line cells is reported and the necessary constructive parameters are realized at 4GHz.

1. Introduction

Transformation electromagnetic is proved to be an efficient way of controlling the propagation of electromagnetic waves. This technique provides a methodology to guide the electromagnetic waves by properly adjusting the constructive parameters of a medium [1]. Based on this new idea, some novel applications are introduced such as cloaks, concentrators, superabsorbers and superscatterers [2]. A method for converting the cylindrical to plane wave has been introduced in [3] by using a full spatially dependent constructive parameters. A similar method of wavefront conversion was applied for designing highly directive antennas [4,5]. In this paper, we report a wavefront conversion technique for TE waves with only a single spatially varying entry in the permeability tensor. Then a loaded two-dimensional (2D) microstrip line network design is reported for implementation purposes.

2. Optical Transformation

The optical transformation sketch for wavefront conversion is shown in Fig.1(a). For changing the wavefront from cylindrical to planar, the concentric circles in the virtual space (x', y') should be mapped to vertical lines in the real space (x, y) with the functions

$$x = f(\rho') = f(\sqrt{x'^2 + y'^2}), \qquad y = g(\varphi') = g(\arctan(\frac{y'}{x'})), \qquad z' = z. \tag{1}$$

Based on the metric invariance of Maxwell's equations [1], the ideal relative constitutive tensors of the rectangular region (the conversion region) in the real space while using the transformation relations in (1) is expressed as the diagonal tensors

$$\bar{\bar{\mu}}^i = \bar{\bar{\varepsilon}}^i = diag\left[\rho' \frac{\partial f}{\partial \rho'} \Big/ \frac{\partial g}{\partial \phi'},\ \frac{\partial g}{\partial \phi'} \Big/ \left(\frac{\partial f}{\partial \rho'}\rho'\right),\ \rho' \Big/ \left(\frac{\partial f}{\partial \rho'}\frac{\partial g}{\partial \phi'}\right)\right] \tag{2}$$

where all the tensor entries are spatially varying in general. However the spatial dependence of these parameters can be reduced by the fact that the wave trajectory in real space remains unchanged for the TE waves, provided that $\varepsilon_z^i \mu_x^i$, $\varepsilon_z^i \mu_z^i$ are held constant [6]. Therefore, one can reduce ε_z^i to a constant κ, then the reduced permittivity entries are scaled by the factor ε_z^i / κ, where the reduced parameters are given by

$$\mu_x = \rho'^2 / \left[\kappa \left(\frac{\partial g}{\partial \phi'} \right)^2 \right], \qquad \mu_y = 1 / \left[\kappa \left(\frac{\partial f}{\partial \rho'} \right)^2 \right], \qquad \varepsilon_z = \kappa . \tag{3}$$

Notice that the selection of the f and g functions can further simplify the spatial dependence of the permeability. Thus we select $\partial g/\partial\phi'$ and $\partial f/\partial\rho'$ as constants where f and g functions are defined as $x = w\rho'/R$ and $y = L\phi'/\pi$ respectively. Here we transform a half circle with radius R to a rectangle with the height L and the width w. Now μ_x is a function of x only and the other constructive parameters of interest (μ_y, ε_z) are spatially invariant. The plane wave propagating along $+x$ direction at the conversion region's edge should be matched to the free space. Then the impedance $Z^{+x} / \left(\sqrt{\mu_0 / \varepsilon_0}\right) = \sqrt{\mu_y / \varepsilon_z} = 1$, as a result $\kappa = R/w$. Finally the constitutive parameters are determined as

$$\mu_x = x^2 R\pi^2 / wL^2 , \qquad \mu_y = R / w , \qquad \varepsilon_z = R / w . \tag{4}$$

In Fig. 1(b), the constructive parameter are plotted for the primary (ideal), reduced and step-wise approximated parameters (for implementation purposes) when $L = 2R = 2w$.

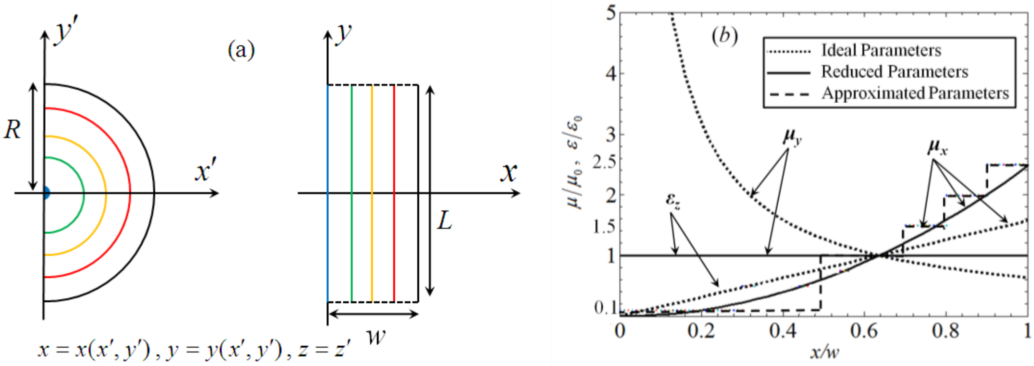

Fig. 1. a) Optical Transformation for cylindrical to plane wave conversion b) The required relative permittivity and permeability for ideal, reduced and step-wise approximated parameters

3. Design and Simulation

To numerically verify the concept, a 2D simulation of TE waves (z-directed E-field) is done (with COMSOL, using FEM). The conversion region comprises layers with the approximated constitutive parameters shown in Fig. 1(b). Fig.2 shows the time domain E-field at 4GHz with and without the conversion region. It is clear that the wavefront is converted from the cylindrical to planar by using the approximated constructive parameters.

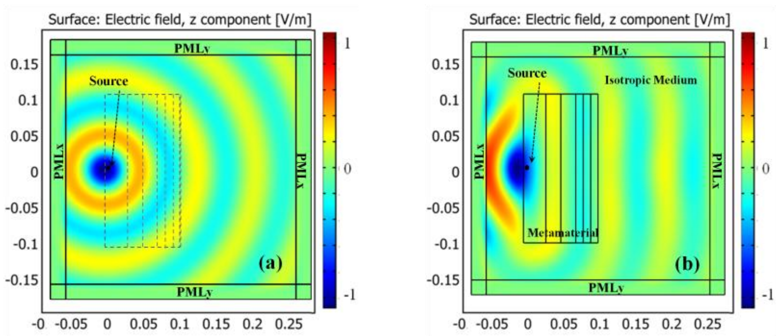

Fig. 2. Electric field distribution in z direction for z-polarized TE, a) the isotropic medium b) converter slab

For implementation purposes, a 2D microstrip network is utilized. In this method, the transmission line grid is loaded with reactive elements to achieve the desirable permittivity and permeability. In Fig. 3, the loading micro-strip elements, the interdigital capacitors and the meander line inductors, are illustrated.

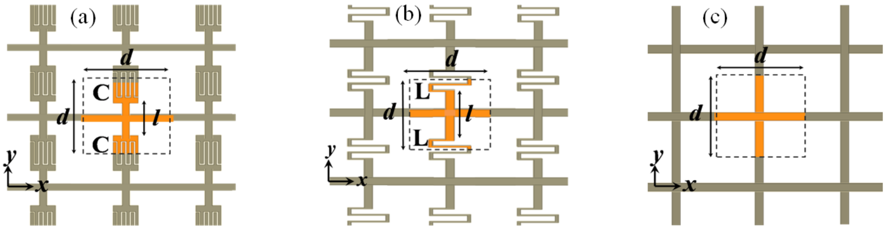

Fig.3. Three types of transmission line unit cells a) capacitor loaded transmission line for $\mu_x < 1$ b) inductor loaded transmission line for $\mu_x > 1$ c) isotropic unit cell for the isotropic medium (free space)

The permeability and permittivity relations of a loaded 2D transmission line network, as in [6],

$$\mu_x = \frac{l}{d} L_{dis} + \frac{Z_{series}}{j\omega d} \qquad \mu_y = L_{dis} \qquad \varepsilon_z = C_{dis} + \frac{l}{d} C_{dis} + \frac{Y_{shunt}}{j\omega d} \tag{5}$$

where L_{dis} and C_{dis} are the distributed inductance and capacitance of the pure microstrip line respectively, *d* is the meshsize, *l* is the length of the microstrip line in the *y* direction shown in Fig.3, Z_{series} models the interdigital capacitor (for $\mu_x < 1$ regions, Fig.3(a)) or the meander line inductor (for $\mu_x > 1$ regions, Fig.3(b)), Y_{shunt} represents the shunt capacitance created by the series load geometries. However, we reduced the effect of the shunt capacitance to a minimum by etching the ground below the series elements. The isotropic region is the unloaded TL network as shown in Fig.3(c). The substrate is chosen as RO4003 ($\varepsilon_r = 3.55$) with the thickness $h = 0.76$ mm. The network's meshsize, *d*, is 6mm and the microstrip line width is 0.5mm. Based on these parameters, the relative permittivity μ_x that can realize the approximated values in Fig. 1(b) is plotted in Fig.4(a) versus the meander line inductor dimensions and in Fig.4(b) versus the interdigital capacitor dimensions.

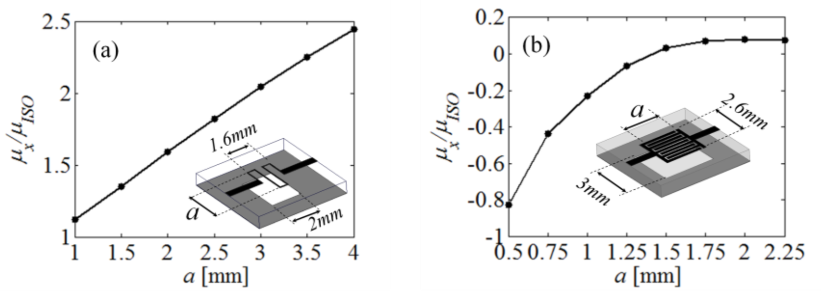

Fig.4 . Full wave simulation results for relative μ_x for a) inductor-loaded cell b) capacitor-loaded cell

4. Conclusion

Optical Transformation is applied for cylindrical to planar wave front conversion. The transformation formulas are simplified such that only the permeability tensor entry μ_x is a function of position, whereas all other parameters are constant. The series-loaded transmission line is proposed for realizing the spatially varying μ_x where the loading interdigital capacitors and the meander line inductors are characterized.

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