Nonlinear cloaking of nanowires

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Abstract

In this paper we study scattering properties of multi-layer plasmonic nanowires with a nonlinear response. With a proper choice of parameters we can either increase or decrease the scattering cross-section by changing the intensity of the external field. We demonstrate the efficient control and even the possibility to reverse the scattering direction by means of small variations of the incident wave amplitude.

1. Introduction

The interest in the studies of invisibility cloaking was stimulated by the pioneering work of John Pendry et al. [1], and it was further studied both theoretically and experimentally [2]. The idea of cloaking, i.e. of creating a special type of cover which bends the radiation around the concealed object, is based on the transformation invariance of the Maxwell's equations. Mathematically, the design of the cloak is relatively straightforward, with simple expression defining the required material parameters of the cloak related to the metric of the virtual space transformation. However, experimentally the predicted parameters are very hard to achieve, especially for higher frequencies.

This problem further studies into simpler ways to achieve cloaking. For hiding small subwavelength particles in rather wide frequency band, Alu and Engheta suggested to coat particles with several layers of various materials [3]. An alternative approach presented in Ref. [4] proposes a different way for reducing the scattering cross-section, namely to cover a spherical nanoparticle with a layer of zero-epsilon material in order to shield it from the radiation, and then suppress the scattering with one more dielectric layer. The remarkable property of such a cloak is that the electromagnetic energy stored in the layer with $\varepsilon = 0$ grows with decreasing coating layer thickness. It means, that ideally in the lossless case, the infinitesimally thin layer contains infinite energy and sustain infinite electromagnetic field amplitudes. The losses attenuate this effect, however the significant field enhancement is still observed for realistic absorption coefficients. This suggests that nonlinear effects can be significantly enhanced in such multi-shell plasmonic nanoparticles, especially in zero-epsilon region. In the following, we consider the effects of nonlinear scattering of the radiation on the core-shell cylindrical particles of this type.

The results of the linear analysis of cylindrical geometry shows that in order to obtain zero dipole scattering one needs to cover the shielded wire with a shell having dielectric permittivity

$$\varepsilon_3 = (R_3^2 + R_2^2)/(R_3^2 - R_2^2) \,, \tag{1}$$

where R_2 and R_3 are the inner and the outer radii of this shell. These results are obtained in a similar way as those in Ref. [4], and verified by numerical calculations, in which we use the multipole expansion of the radiation in cylindrical Mie modes.

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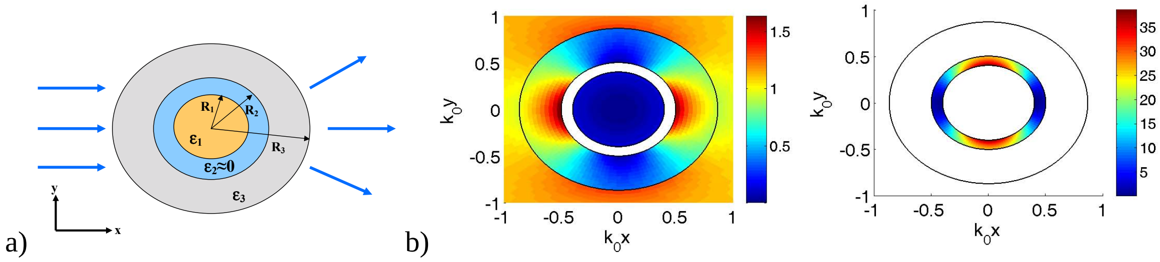

Fig. 1: a) Geometry of the problem; b) Distribution of the intensity of the electric field in the whole space except layer 2(left panel) and in layer 2 only (right panel).

2. Numerical simulations.

We consider scattering of the TM polarized plane wave on the core-shell cylindrical structure as shown in Fig.1. The magnetic field in the incident wave is $h_z = H_{inc} \exp(i\omega t)$ has only z-component and it can be represented as a sum of Bessel functions,

$$H_{inc} = A_0[J_0(\rho) + 2\sum_{m=1}^{M} i^m J_m(\rho)\cos(m\phi)] \, , \qquad \rho = k_0 r \, . \tag{2}$$

Inside the structure, each layer j have the dielectric constant of ε_j, and the magnetic field $H^{(j)} \exp(i\omega t)$ is represented in the following way

$$H^{(j)} = \sum_{m=0}^{M} [A_m^{(j)} J_m(\rho_j) + B_m^{(j)} Y_m(\rho_j)] \cos(m\phi) \, , \tag{3}$$

where J_m and Y_m are Bessel functions of the first and the second kind, $\rho_j = k_0\sqrt{\varepsilon_j} r$, and summation goes over finite (but large enough) number M of azimuthal harmonics. The continuity conditions of magnetic and tangential (ϕ-component) electric fields provide the relations between the amplitudes $A_m^{(j)}$, $B_m^{(j)}$ in j-shell and $A_m^{(j+1)}$, $B_m^{(j+1)}$ in $j+1$-shell. These conditions can be written in the matrix form

$$\hat{S}_m^j \mathbf{V}_m^j = \hat{S}_m^{j+1} \mathbf{V}_m^{j+1} \text{ , or } \mathbf{V}_m^{j+1} = (\hat{S}_m^{j+1})^{-1} \hat{S}_m^j \mathbf{V}_m^j \, ,$$

where the components of vector $\mathbf{V}_m^j$ are the amplitudes: $\mathbf{V}_m^j = (A_m^j, B_m^j)$. In the core ($j = 1$) the expansion (3) includes only J_m-terms because of finite field value at $r = 0$, and the amplitude $A_m^{(1)}$ is defined from the condition that the field outside the structure is a superposition of incident wave (2) and cylindrical outgoing wave (Hankel function of the second kind). As a result, the linear problem is solved by calculating inverse matrices, with the algorithm being similar to that of the transfer matrix method. The obtained linear solution shows highly resonant character of the field intensity in the zero-epsilon shell (see Fig.1b).

Nonlinear problem is solved by using the iterative scheme. We find the radial and azimuthal multipole components of the electric field in nonlinear shielding layer by means of direct solution of differential equation. The sum over the multipole contributions gives us the Kerr nonlinear perturbation of dielectric permittivity as a function of radial coordinate and azimuthal angle. Then, we decompose the nonlinear term into a series of $\cos(m\phi)$. The linear shells are calculated as before by means of matrix method. As a result we find the multipole scattering amplitudes. The procedure is iterated until the amplitudes do not change with new iterations.

3. Numerical results.

First, we consider the linear scattering properties of multishell nanowire with the shell sizes $k_0R_1 = 0.4$, $k_0R_2 = 0.5$, $k_0R_3 = 0.866$ and shell dielectric permittivities $\varepsilon_1 = 15$, $\varepsilon_2 = 0.1 + 0.02i + \alpha|\mathbf{E}|^2$,

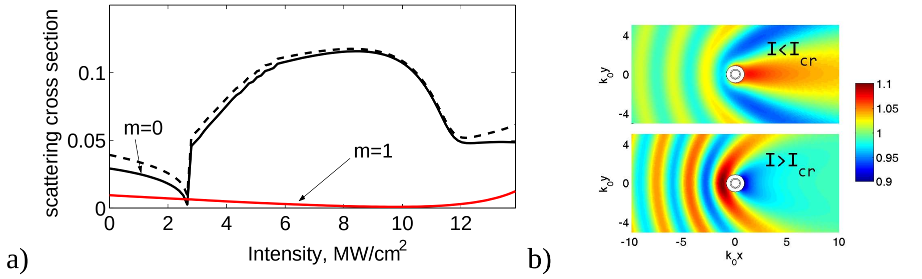

Fig. 2: a) Normalized scattering cross-sections for several lower order multipoles (solid lines, m = 0; 1), as well as total SCS (dashed) as functions of the intensity; b) Distribution of the scattering electric field for two different values of intensity: 2.1 MW/cm^2 (upper panel) and 2.66 MW/cm^2 (lower panel).

$\varepsilon_3 = 2$, which satisfies the condition (1), but which is slightly out of the resonance in the shielding layer ($\varepsilon_2 \neq 0$). We suppose that the resonance media of this layer has Kerr-type defocussing nonlinearity with nonlinear coefficient $\alpha \sim -5 \cdot 10^{-8}$ ESU [5]. Varying R_3 and ε_3 we have shown that indeed the lowest scattering cross-section (SCS) in linear regime corresponds to the case, when condition (1) is satisfied. However, in contrast to the spherical case, it turns out that zero-order azimuthal harmonic ($m = 0$) plays a significant role in scattering, and this symmetric mode causes nontrivial scattering behavior of the structure in nonlinear regime.

For low power, the core-shell particle scatters mostly in the forward direction, however with increasing power, the scattering changes to the backward direction. This transition coincides with the sharp increase of the SCS of the m=0 mode, shown in Fig. 2(a) for the intensity value of $I_{cr} \approx 2.7$ MW/cm^2. Further increase of the intensity makes scattering more unidirectional-like until the second threshold, when the scattering becomes mostly forward again. Figure 2b shows the distribution of the field amplitudes for different incident power intensities. It can be clearly seen that the overall scattering strength is dramatically changing, with a large variation of the directivity.

4. Conclusions

We have studied the wave scattering by nonlinear multi-shell nanoparticles and demonstrated that the cloaking efficiency of the particles can be controlled by varying the intensity of the incident wave. The nonlinear response of such core-shell particles is enhanced significantly in the layer with near-zero dielectric permittivity. We have revealed that the scattering direction can be abruptly changed by the incident wave due to the energy exchange between the multipole modes of the structure.

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