Chirp as a tool to control the propagation velocity of a femtosecond laser pulse in negative index metamaterials

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Abstract

We study theoretically propagation of a femtosecond laser pulse in a negative-index metamaterial (NIM). Our results show that at a certain wavelength range one can control the propagation velocity of the pulse simply by changing its initial chirp.

1. Introduction

In recent years NIMs attract the great attention due to their exotic properties in response to the electromagnetic waves [1] and their promising applications [2] as a novel optical element. NIMs are highly dispersive media and group velocity of laser pulse in NIMs can be significantly smaller (slow light) or greater (fast light) than that in vacuum [3, 4]. We show that, at a certain wavelength range, one can control the propagation velocity of a femtosecond laser pulse in NIM simply by changing its initial chirp [5]. The underlying mechanism of such control is related to the wavelength-dependent absorption [6] in NIMs.

2. Results and Discussion

NIMs consist of structural units with a dimension smaller than the wavelength of the incident light. This implies that the overall response of NIMs can be well-described by the "effective" (homogenized) permittivity, $\epsilon(\omega)$, and permeability, $\mu(\omega)$.

To be more specific we consider the NIM recently reported by García-Meca *et al.* [7]. We used Drude model [2] for permittivity and permeability and made parameter fittings to reconstruct complex refractive index n of this NIM. Fig. 1 shows real n' and imaginary n'' parts of obtained n.

Once the complex refractive index has been obtained the group and phase velocities defined by $v_g(\omega) = c_0/[n'(\omega) + \omega\partial n'(\Omega)/\partial\Omega|_{\Omega=\omega}]$ and $v_p(\omega) = c_0/n'(\omega)$ can be easily calculated as a functions of wavelength $\lambda = 2\pi c_0/\omega$. Fig. 1 shows that there are four regions where v_g and v_p have different signs [3]. In all four regions, the Poynting vector is always positive, i.e. directed along the positive z direction.

Assuming that the incident pulse is well-approximated by the plane wave and the NIM is a bulk (i.e., no interfaces and backscattering) we employ the Helmholtz equation to describe the propagation. Its solution of the field amplitude $E(z,t)$ of laser pulse in the moving frame ($t \to t - z/c_0$ where c_0 being the speed of light in vacuum) is written as [8],

$$E(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \widetilde{E}(0,\omega) e^{i\left[\beta(\omega)-\frac{\omega}{c_0}\right]z} e^{-i\omega t} d\omega, \tag{1}$$

where $\widetilde{E}(0,\omega)$ is the initial spectral amplitude and $\beta(\omega) = [n(\omega)\omega]/c_0$ is a complex dispersion function.

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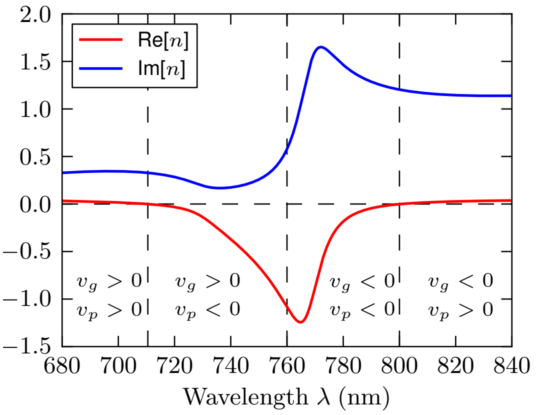

Fig. 1: Real and imaginary parts of the refractive index, n' and n'' as a function of wavelength.

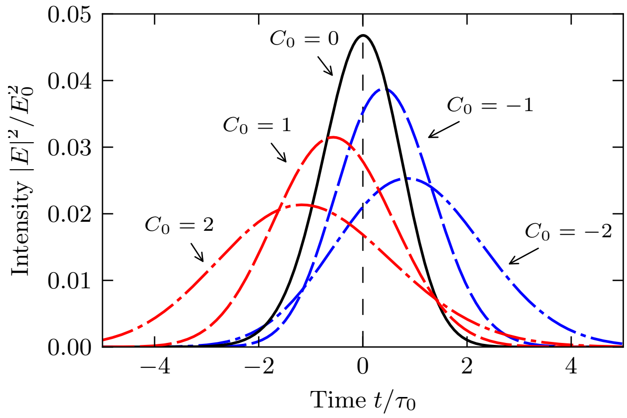

Fig. 2: Normalized intensity profile of the pulses, $|E(z,t)|^2/E_0^2$, with $\tau_0 = 100$ fs and $\lambda_0 = 765$ nm for different chirp parameters, C_0, after propagating the distance of 185 nm in the NIM.

We assume incident pulse at a central frequency of ω_0 (central wavelength $\lambda_0 = 2\pi c_0/\omega_0$) to be Gaussian and linearly chirped with spectral amplitude, $\widetilde{E}(0,\omega)$, given by:

$$\widetilde{E}(0,\omega) = \sqrt{2\pi}\tau_0 E_0 \exp\left[-\frac{\tau_0^2(\omega-\omega_0)^2}{2}(1-iC_0)\right]. \tag{2}$$

where E_0 and C_0 are the initial field amplitude and dimensionless chirp parameter, respectively, and τ_0 is the durations of the transform-limited pulse.

We consider laser pulse with $\tau_0 = 100$ fs ($\simeq 166$ fs FWHM as in Ref. [3]), and $\lambda_0 = 765$ nm. From the numerical integration of Eq. (1) with different chirp parameters we calculate the intensity profile of the pulse after the propagation distance of 185 nm. The results are shown in Fig. 2. Interestingly the group delay of the laser pulse can be negative or positive, and hence its propagation velocity can be subluminal or superluminal, depending on the initial chirp.

To investigate the dependence of the group delay on the initial chirp at different central wavelength, we numerically integrate Eq. (1) for laser pulse with $\tau_0 = 100$ fs and different C_0 and λ_0. The result is shown in Fig. 3(a). We can classify Fig. 3(a) into seven regions; I, V, and VII where the initial chirp has very little effect; II and III where only the specific sign of C_0 leads to the negative group delay; the most interesting regions IV and VI where the sign change of C_0 results in the sign change of group delay. Namely, in the spectral regions IV and VI, switching between the subluminal and superluminal propagation is possible simply by changing the initial chirp.

To better understand Fig. 3(a) we now look into the analytical solution of Eq. (1). For that purpose we do the Taylor expansion of dispersion function $\beta(\omega)$ about ω_0. For convenience in what follows we rewrite $\beta(\omega)$ as $\beta(\omega) = k(\omega) + i\Gamma(\omega)$, where $k(\omega)$ and $\Gamma(\omega)$ represent the wave number and absorption coefficient, respectively. We expand $k(\omega)$ up to the second order and $\Gamma(\omega)$ up to first order. Then the analytical solution of Eq. (1) gives the following expression for group delay $\triangle t_g$:

$$\triangle t_g = \triangle t_{g0} + \Gamma_1 C_0 z, \tag{3}$$

where $\triangle t_{g0} = [1/c_0 - 1/v_g(\omega_0)]z$ is the group delay of transform-limited pulse and $\Gamma_1 = \partial\Gamma/\partial\omega|_{\omega_0}$.

In contrast to the group delay the phase delay does not depend on the initial chirp, which was confirmed by our numerical results.

In Fig. 3(b) we show the results of the group delay calculated by Eq. (3), which is to be compared with Fig. 3(a). The agreement is very good. This finding indicates that the wavelength-dependent absorption, $\Gamma(\omega)$, is very important to correctly describe the propagation dynamics for the chirped incident pulse.

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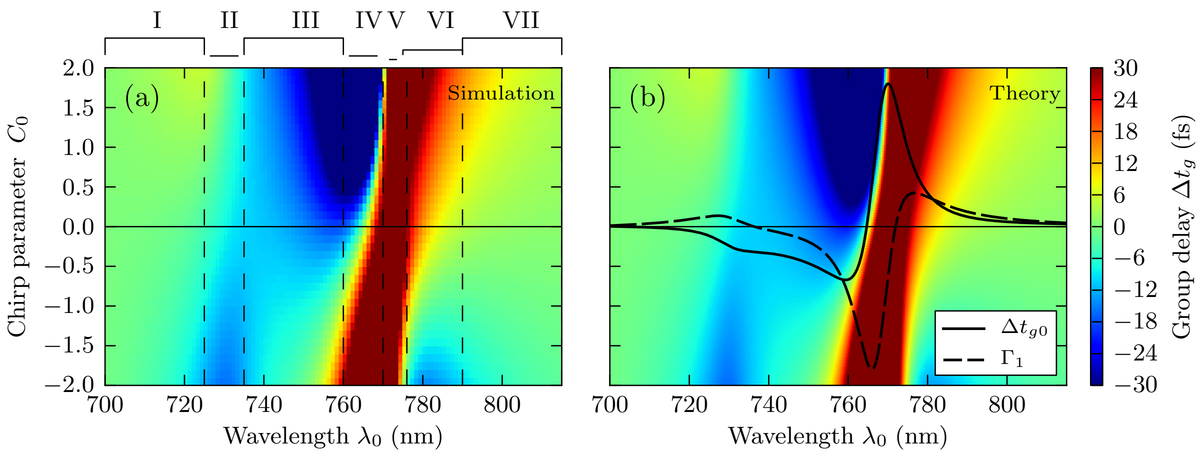

Fig. 3: Group delay, $\triangle t_g$, of the pulse with $\tau_0 = 100$ fs after propagating of 185 nm in the NIM as function of central wavelength and chirp parameter. (a) numerical integration of Eq. (1) and (b) analytical solution by Eq. (3). Solid and dashed curves in graph (b) represent the normalized values of the group delay, $\triangle t_{g0}$, for the transform-limited pulse and the first derivative of the absorption coefficient, Γ_1.

We can explain the chirp-dependent group delay in the NIM shown in Fig. 3 in a more intuitive way: Recall that the instantaneous frequency of the linearly chirped pulse changes in time. For example, if the pulse is positively chirped ($C_0 > 0$) the red frequencies are followed by the blue ones. Due to the wavelength-dependent absorption coefficient, $\Gamma(\omega)$, some frequency components are absorbed more than the others, and accordingly the leading or trailing edge of the pulse is more absorbed during the propagation. As a result the center of gravity of the pulse and accordingly the maximum of the pulse envelope is shifted in time, leading to the chirp-dependent group delay, as Eq. (3) shows.

4. Conclusion

In conclusion we have theoretically studied the propagation dynamics of a femtosecond pulse in the NIM. Using the realistic parameters to represent the NIM from the recent work [7], we have found that the introduction of the initial chirp significantly alters the propagation in the NIM, and the control of the propagation velocity (superluminal or subluminal) is possible simply by changing the initial chirp. Our results indicate that the NIMs can be used for an extremely compact device to control the propagation velocity of a femtosecond pulse.

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