

## Subwavelength imaging with materials of in-principle arbitrarily low index contrast

Y.G. Ma<sup>1</sup>, S. Sahebdivan<sup>2</sup>, C.K. Ong<sup>3</sup>, T. Tyc<sup>2,4</sup> and U. Leonhardt<sup>2</sup>

<sup>1</sup>Center for Optical and Electromagnetic Research, State Key Laboratory of Modern Optical Instrumentation, Zhejiang University, Hangzhou 310058, China,

<sup>2</sup>School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews KY16 9SS, UK, <sup>3</sup>Centre for Superconducting and Magnetic Materials, Department of Physics, National University of Singapore, Singapore, 117542, Singapore,

<sup>4</sup>Faculty of Science and Faculty of Informatics, Masaryk University, Brno, Czech Republic. Email: yungui@zju.edu.cn and ulf@st-andrews.ac.uk

## Abstract

Perfect imaging with Maxwell's fish eye opens the exciting prospect of passive imaging systems with a resolution no longer limited by the wave nature of light. But it also challenges some of the accepted wisdom of super-resolution imaging and therefore has been subject to controversy and discussion. Here we discuss an idea for even simpler perfect-imaging systems based on geometrical optics and prove by experiment that it works.

The archetype of perfect imaging with positive refraction is Maxwell's fish eye [1]-[6]. It is easy to understand why Maxwell's device may focus light with unlimited precision if we consider the fact [2] that it represents the geometry of a virtual sphere — in physical space light propagates in a medium with Maxwell's profile as if it were confined to the surface of a sphere. Imagine light emission on the virtual sphere. A wave emitted at a point source on the sphere must converge with point-like precision at the antipodal point, simply due to the perfect symmetry of the sphere. The sphere thus turns the wave expanding from the point of emission into a converging wave at the image point.

It is also easy to understand why perfect imaging is possible in this case, using an argument by Feynman [7] that goes as follows [8]. Maxwell's equations of light propagation are completely time-reversible. Therefore the emission process at a point source can be completely reversed at the image, provided one crucial element is present, a source run in reverse — a drain. Otherwise the absorption at the image would not be the exact time reverse of the emission at the source. The drain at the image point is also something natural in imaging where one wishes to detect an image, for example by photochemical reactions or in a CCD array. The drain represents a detector.<sup>1</sup> The crucial point of perfect imaging is that, given a choice of detectors in the image area, the light localises at the correct ones.

The fact that the correct light localisation naturally happens in Maxwell's fish eye is also understandable if we imagine the absorption in an array of detectors as the time reverse of the emission by a collection of point sources. Given perfect time symmetry, the light must settle down at the image points that correspond to the actual source points and avoid the ones corresponding to potential source points that did not emit. In this way, a sharp image is formed with a resolution given by the cross section of the detectors and not by the wave nature of light.

Despite the simplicity and generality of these ideas and arguments, they stirred up controversy [11]-[23] because they challenge some of the accepted wisdom of super-resolution imaging [24]. It has been an

<sup>&</sup>lt;sup>1</sup>The localisation of a wave by a time-reversed emission process was beautifully demonstrated with sound waves using an active drain [9]. Note that an active drain is not necessary: in our previous microwave experiment [10] we used a passive drain — the drain only responded to the incident radiation — and obtained subwavelength resolution.



often-unquestioned mantra that perfect imaging is only possible if evanescent waves are amplified in some way. It has been firmly accepted that evanescent waves are the carriers of subwavelength details and that the near-field information is lost in the far field, as evanescent waves exponentially decay away. However, this belief is in contradiction with Feynman's general argument based on the time-reversal symmetry of Maxwell's equations [7, 8]. The near field of a point source is not lost in the far field, but rather constitutes a natural part of the emission process. If the emission is reversed in a point detector the near field naturally arises. The precise information about the point of emission is not gone, it is carried by the light wave; yet to retrieve it requires unusual imaging systems [25].

In a previous paper [10] we implemented Maxwell's fish eye for microwaves and observed subwavelength imaging. Here [26] we demonstrate an idea [25] for a modified fish-eye mirror that requires, in principle, an arbitrarily low index contrast. Such a device can be made with conventional graded-index materials. In our case, however, we used a microwave metamaterial, because of its low cost. We have demonstrated subwavelength imaging for microwaves, because microwave technology gives as a degree of detail and precision that is currently impossible in the optical range of the spectrum. In addition to demonstrating an idea for reducing the demands on the materials required for perfect-imaging systems, our experiment illustrates two fundamental points that may also become important in practice.

First, we demonstrate that it is not necessary to detect the field with perfect efficiency. The undetected part of the field does not localise at the detectors with subwavelength resolution, but the detected part of the field finds its way into the right detectors. The subwavelength image appears in the detectors, but not necessarily in the field around them. However, what counts in practice is only the detected part of the field, which implies that the efficiency of the detectors is less important than their resolution.

Second, for the perfect imaging of light waves ray optics seems sufficient; we only require that all light rays from any point of the source intersect at the corresponding image point, which is the defining property of an absolute optical instrument [3]. Within geometrical optics, all rays reach the image with the same phase [3], which seems to guarantee perfect imaging also in wave optics.



Fig. 1: Modified fish–eye mirror. Left: light emitted from a point outside the focusing index profile. Not all rays are focused at the image point. Right: perfect imaging within ray optics if the light is emitted inside the index profile.



Fig. 2: Imaging experiments. (a): schematic diagram of the experiments. Two sources are placed on the yellow dashed circle. The distance between the sources is  $0.2\lambda$  where  $\lambda$  is the local wavelength. An ordinary instrument would not be able to resolve the two sources, but here we image them with a modified fish-eye mirror (Fig. 1). The image is detected by 9 outlets also indicated on the yellow dashed circle. We average over 4 runs of the experiments and obtain statistical error bars. (b): imaging outside of the lens, which corresponds to the left picture of Fig. 1 where we do not expect perfect imaging. We record the throughput through the outlets (black dots) and scan the field along them (grey dots). Our measurements show that the two sources are not resolved. (c): imaging within the lens where perfect imaging is possible. The scanned field (grey dots) does not exhibit marked peaks at the expected image points, but the throughput (black dots) distinguishes the two sources, thus demonstrating subwavelength imaging with a modified fish-eye mirror.

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