Field Transformation: A Paradigm for Designing Wide Band, Wide-Angle Dual-Polarized Lenses and Cloaks with Physically Realizable Materials

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Abstract

In this paper, we review the Transformation Optics (TO) approach to designing cloaks, superlenses and identify some of the difficulties associated with the practical devices realized by using this approach, for instance polarization dependence, physical realizability and dispersion. Following this we present an alternative method based upon Field Transformation (FT), as a way to mitigate some of these problems.

1. Introduction

Transformation Optics (TO) is a relatively new field, which has caught the attention of many workers in recent years, such as Pendry [1], Leonhardt [2], Smith [3], Hao [4], Werner [5] and others [6-10]. One of the attractive features of TO is that it offers a new way of designing cloaks, superlenses and directive antennas. For instance, to follow the TO paradigm to design a cloak for a cylindrical rod that renders the structure invisible, we would simply transform the original cylinder whose radius is a into another which has a smaller radius, say b (see Fig.1), which resides in the virtual domain, and is surrounded by free space. If we now let the cylinder in the virtual domain become vanishingly small, i.e., let $b \rightarrow 0$, its scattering cross-section would follow suit and, consequently, the cylinder would become invisible and would not be seen by an incoming wave that impinges upon it, provided it is surrounded by free space. Of course, our task is to make the original cylinder (radius a) mimic the same behavior, as that of the one in the transformed system, and we turn to TO to design a cloak, as shown in Fig.1 (a), to make this happen. What TO tells us is that we can do this by transforming the geometry in Fig.1 (b) into the one in Fig.1 (a), and determining the material parameters (ε, μ) of the cloak such that the fields in the two systems are the same. TO also tells us that the ε and μ in the system in the left (Fig.1.a) are related to the ones in the system in the right (Fig.1.b) via the relationship

$$\epsilon' = (\mathrm{J} \cdot \epsilon \cdot \mathrm{J}^{\mathrm{T}})/det\ (\mathrm{J}) \quad (1)$$

$$\mu' = (\mathrm{J} \cdot \mu \cdot \mathrm{J}^{\mathrm{T}})/det\ (\mathrm{J}) \quad (2)$$

where J is the Jacobian of the transformation system.

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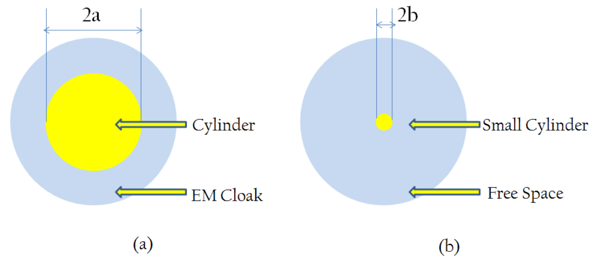

Figure 1: (a) The original and (b) transformed geometries.

The process we described above is very straightforward. The steps we follow to design the cloak for a given object are:

(i) Choose an auxiliary object, which has the desired scattering characteristics when embedded in free-space. For an ideal cloak, we want the characteristics to be such that it renders the target invisible.

(ii) Transform the geometry of the auxiliary object into that of the original target for which we are attempting to design the cloak. When the original target has an arbitrary shape, and we choose the transformed object to be of the same shape—except smaller in scale—then the Jacobian can be derived relatively simply; otherwise, numerical work would be required, unless the geometrical relationship between the two objects enables us to handle the transformation between the two objects analytically.

(iii) Next, design the cloak by using the formula (1, 2), having derived the Jacobian matrix relating the two systems in the previous step.

On the face of it, the procedure described above is simple to understand and easy to follow. However, there is an important practical issue, which we have skirted thus far, and which we must address before we can fabricate the cloak. This has to do with the physical realization of the materials of the cloak, whose parameters have been dictated by the TO algorithm. Specifically, we ask: How do we realize the ε and μ values that are not naturally available, especially when these parameters are less than unity, and/or they have unrealistically large values? A related question is: How can we synthesize materials with the desired $[\varepsilon]$ and $[\mu]$ that are anisotropic in nature and have tensor properties, which further exacerbates the realizability problem?

A short answer to the above questions is: There are no natural materials that satisfy these requirements and our only option is to artificially synthesize them, if possible, conceding the fact the cloak would be narrowband, lossy, dispersive, polarization-dependent, as well as dependent upon the angle of the incident wave.

2. Cloak and Lens Design Using the Field Transformation Approach

We will begin the presentation by providing a simple interpretation of Field Transformation (FT) approach by using the integral form of Maxwell's equations,

$$\oint E \cdot dl = -\frac{\partial}{\partial t}\iint B \cdot dS \tag{3}$$

$$\oint H \cdot dl = \frac{\partial}{\partial t}\iint D \cdot dS \tag{4}$$

for an arbitrary geometry, and using scaling to define the auxiliary geometry.

We will show how we can relate the material parameters of the two systems—the original and the transformed (or virtual)—by using (3) and (4), and why we must require unrealistic material parameters when we try to design an invisible cloak for the original geometry by scaling down that geometry by a very large factor.

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Next, we will present the fundamental concepts of the Field Transformation approach, which attempts to overcome the above difficulty. The concept is depicted by Fig.2, where we transform the fields from the input to the output aperture by choosing the parameters of the intervening medium and tracking the propagation of the fields through this medium to realize the desired magnitude and phase behaviors of the fields in the output aperture.

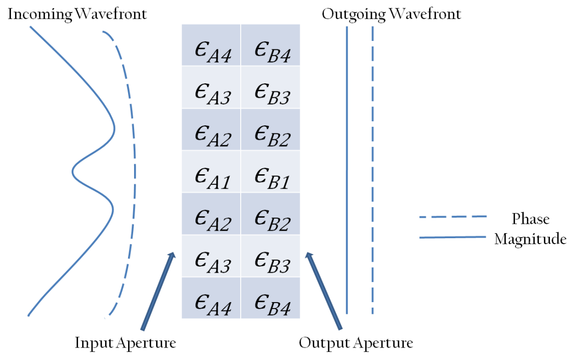

Fig. 2: Transforming the magnitude and phase of the specified fields in the input aperture into the desired distribution in the output aperture by using multilayered materials.

Following this, we will present some examples where we apply the FT approach to design lenses, as well as absorbers for RCS reduction.

For the lens problem we will illustrate the FT approach by considering two cases: (1) Graded index lens synthesized by using a low-loss artificial dielectric, which works over the entire Ku band, and which handles both polarizations equally well. We also present an FT-based design that utilizes real dielectrics and also has the desired properties listed above, namely wide bandwidth and low loss. We should mention here that we will use the FT along with the Genetic Algorithm (GA) to implement this type of lens design.

Finally, we will go on to describe a blanket (or absorber coating), as shown in Fig.3, which is designed to reduce the RCS, and which is very broadband. It is designed to covering either the 2-18 GHz or the 4.6-18 GHz band, depending upon the thickness of the absorber which varies from 3 to 10 mm. It also handles both polarizations and arbitrary angles of incidence that are typically beyond the reach of the conventional TO-based designs. The presented design also utilizes only available materials, with complex ε and μ that can be fabricated in the laboratory. The thickness of the absorber is only a few mms and not a few wavelengths as in the conventional design of cloaks. It should be mentioned that the blanket reduces the reflection (>10dB)—both monostatic and bistatic—though only in the reflection region, and not in the forward-scattering region.

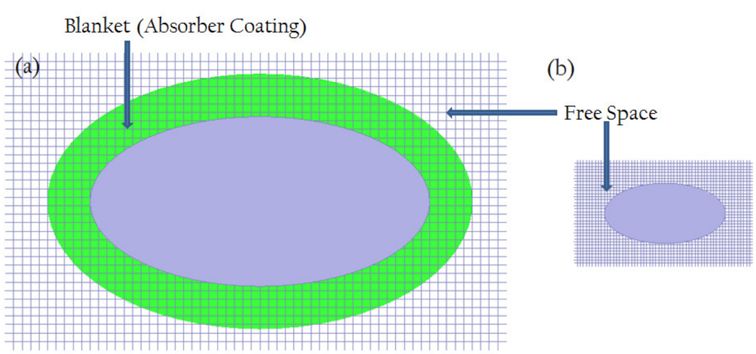

Fig. 3: (a) Original geometry of the arbitrary object and (b) scaled geometry of the object.

Also the absorber is designed to handle arbitrary geometries (Fig.4) and we show how we can combine FT with TO for this purpose.

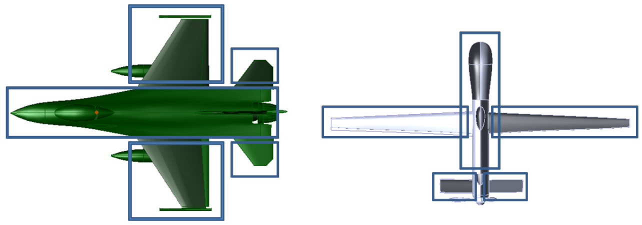

Fig. 4: Arbitrarily-shaped radar target.

3. Conclusion

This paper has presented the Field Transformation (FT) approach as an alternative to Transformation Optics (TO) for designing optical devices, e.g., lens antennas, as well as cloaks. Our focus has been on designing these devices such that they perform over a wide frequency band, are low-loss, and have the ability to handle arbitrary polarization as well as angles of incidence, which the TO-based designs seldom can do. We should reiterate that Transformation Optics (TO) and Field Transformation (FT) approaches are fundamentally different, and it is important to understand that they do not lead to the same designs for devices. Specifically, the FT approach has been developed to circumvent the issues of realizability, narrow bandwidth, polarization sensitivity and anisotropy that are frequently encountered in TO designs.

The presentation will not only point out the fundamental differences between the two approaches, but will also discuss how they could be hybridized for some applications where it may be appropriate and advantageous to do so.

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