Analytical modeling of conformal metasurface mantle cloaks for cylindrical objects

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Abstract

In this work, we demonstrate the concept of mantle cloaking for cylindrical objects using realistic ultra-thin metasurfaces composed of 2-D printed (patches, Jerusalem crosses, and cross dipoles) and slotted structures (meshes, slot-Jerusalem crosses, and slot-cross dipoles). We use a rigorous analytical model based on Lorenz-Mie scattering theory which utilizes the two-sided impedance boundary conditions at the metasurface. Specifically, the analytical expressions for the metasurface surface impedances are borrowed from the analysis of planar grid-array elements. It is shown that, by properly tailoring the surface reactance of the metasurface mantle cloaks, the scattering from a given object can be significantly reduced.

1. Introduction

In recent years the possibility of achieving invisibility has attracted much attention with the introduction of coordinate transformation (CT) cloaking method, put forward by Pendry *et al* [1]. Since then various approaches have been introduced to study the effect of cloaking, such as plasmonic cloaking [2] and cylindrical transmission-line cloaking [3]. Recently, in [4] a different cloaking technique based on the concept of "cloaking by a surface" has been proposed to reduce the visibility of various types of objects, wherein scattering cancellation is obtained using a metasurface, rather than a bulk metamaterial. In these studies, the required surface reactance was obtained by numerically optimizing the shape of the metasurface inclusions.

In this work, on the other hand, we study analytically and verify numerically mantle cloaks for cylindrical objects using various 2-D printed (patches, Jerusalem crosses, and cross dipoles) and slotted structures (meshes, slot-Jerusalem crosses, and slot-cross dipoles). Our method is still based on the mantle cloaking technique proposed in [4], however, here we use a rigorous analytical model to describe the surface impedance of the mantle cloaks with the goal of simplifying the design. The analysis is based on Lorenz-Mie scattering theory, which employs the two-sided impedance boundary conditions at the surface of mantle cloak. The analytical expressions for the grid-impedances of the cylindrical metasurface cloaks (printed and slotted) have been taken from the solution of the plane-wave excitation for planar sub-wavelength grids. This largely facilitates the design of cylindrical cloaks in realizing the required surface reactances that maximally suppresses the overall scattering. Although, our approach of using various sub-wavelength periodic elements has been effective in achieving the cloaking effect, here we consider the case of 2D cylindrical mantle cloaks formed by sub-wavelength slotted and printed Jerusalem crosses.

2. Theory

Consider a transverse magnetic (TM) polarized plane wave normally incident ($\theta = 90^\circ$) on (a) an infinite dielectric cylinder with relative permittivity ε_r = 10 and radius $a = \lambda_0/10$, covered by a mantle

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cloak formed by periodic sub-wavelength slotted Jerusalem crosses with radius $a_c = 1.05a$, $\varepsilon_c = 1$ [see Fig. 1(b)] and (b) an infinite conducting cylinder with radius $a = \lambda_0/10$, surrounded by a cloak formed by the dual structure: printed Jerusalem crosses with radius $a_c = 1.25a$, with the space between the cloak and the cylinder filled with a dielectric of thickness $a_c - a$ and relative permittivity $\varepsilon_c = 10$ [see Fig. 1(c)].

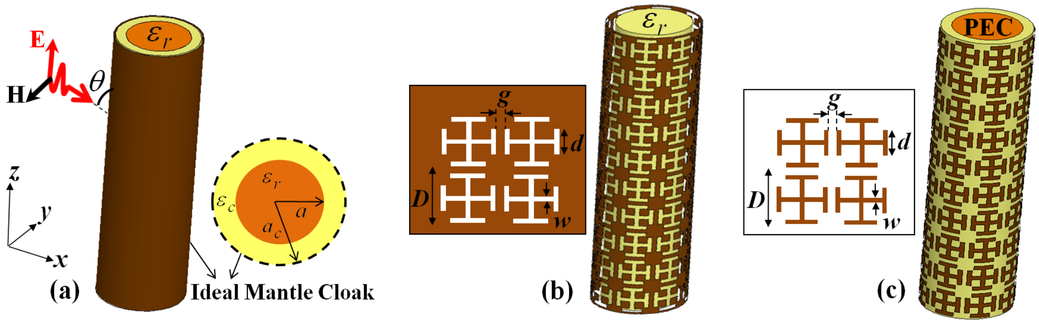

Fig. 1: Schematics of the cylindrical objects covered by various mantle cloaks: (a) an ideal homogeneous and isotropic reactive surface, (b) a conformal array of slotted Jerusalem crosses, and (c) a conformal array of printed Jerusalem crosses.

For any arbitrary incidence angle, the scattering fields can be calculated analytically by enforcing the two-sided impedance boundary conditions on the metasurfaces. The grid impedance expression for a planar array of printed Jerusalem crosses is taken from [5]. For the slotted Jerusalem crosses, the grid impedance expression is obtained by using the approximate Babinet's principle. We verify with full-wave simulations that, due to the sub-wavelength features of metasurface inclusion designs, the grid expressions derived for the planar metasurfaces [5], can still be used to tailor the surface reactances of the metasurface-wrapped cylindrical mantle cloaks. Here we assume lossless metasurfaces, and hence, the impedance is purely imaginary. Applying the well-known Lorenz-Mie scattering theory, the total scattering width (SW), which is a quantitative measure of the visibility of the cylinder, can be easily obtained.

3. Numerical results

First, we consider a dielectric cylinder covered by an ideal mantle cloak characterized by a homogeneous, non-dispersive surface reactance and study the variation of the SW as a function of surface reactance of the mantle cloak for a TM-polarized plane wave at normal incidence ($\theta = 90°$). It is observed that, for some specific inductive values, a significant reduction in the SW is achieved. Then, we consider a realistic design of metasurface patterned with slotted Jerusalem crosses. We optimize the parameters of the grid using the grid-impedance expression in [5] to provide the required inductive surface reactance, which cancels the scattering from the dielectric cylinder. Fig. 2(a) shows the frequency response of the SW for the dielectric cylinder with and without cloak. It can be seen that a significant scattering reduction is achieved at the design frequency f_0, when compared to the uncloaked scenario. The dimensions of the slotted Jerusalem crosses array used in the analysis [see Fig. 1(b)] are: $D = \lambda_0/15$, $w = \lambda_0/200$, $g = \lambda_0/650$, and $d = \lambda_0/22$. Next, we study the variation of SW versus the surface reactance for a conducting cylinder with same size. It is seen that in this case a mantle cloak with capacitive response may significantly reduce the overall scattering. Hence, an immediate choice for this cloak design is the dual design of complimentary metasurface: printed Jerusalem crosses array [see Fig. 1(c)]. The required surface reactance is realized by choosing $D = \lambda_0/8$, $w = \lambda_0/125$, $g = \lambda_0/100$, and $d = \lambda_0/28$. It is clear from Fig. 2(b) that a significant reduction of scattering from the conducting cylinder can be achieved by using the printed Jerusalem crosses mantle cloak, when compared to the case without a proper cloak. Here we note that the effectiveness of the proposed analytical model is validated by the full-wave numerical simulation, as evidently seen in Figs. 2(a) and 2(b). Figure 3 shows the numerical results of the electric field distribution on the H-plane for the cloaked and uncloaked cases for both dielectric and conducting cylinders at the design frequency. In the presence of the cloaks, the wavefronts are almost unperturbed even in the near field [Figs. 3(a) and 3(c)]; however for the case without the cloaks, significant distortion of the electric field is caused by the stronger scattering [Figs. 3(b) and 3(d)].

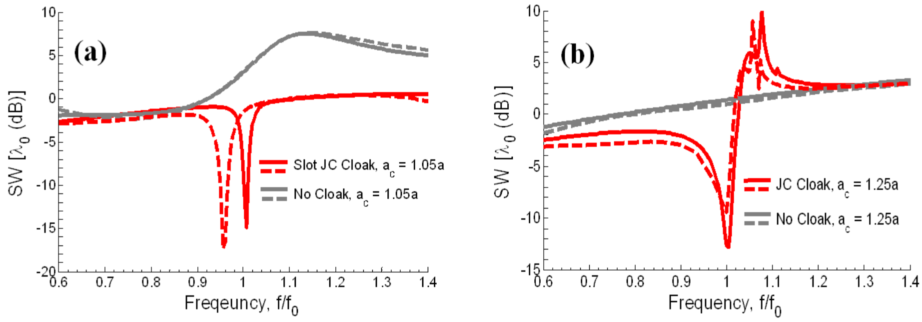

Fig. 2: Variation of the SW versus frequency of operation: (a) of the infinite dielectric cylinder with and without slotted Jerusalem crosses cloak and (b) of the infinite metallic cylinder with and without printed Jerusalem crosses cloak. The analytical results are represented by solid lines and full-wave HFSS [6] results by dashed lines.

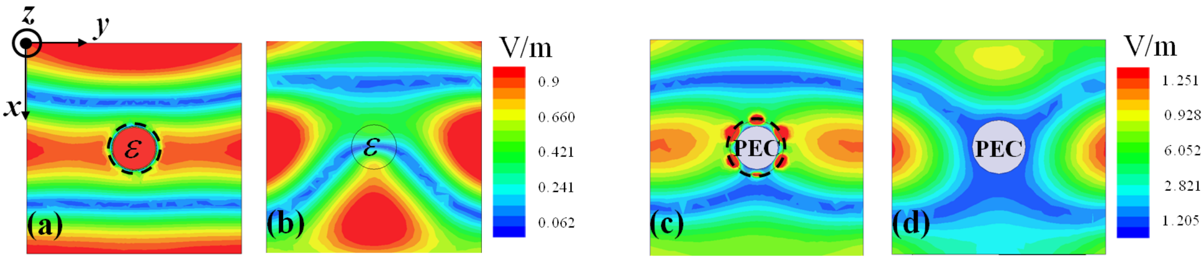

Fig. 3: Snapshots of electric field distributions at the design frequency for a dielectric infinite cylinder (a) with and (b) without the slotted Jerusalem crosses cloak ($a_c = 1.05a$); a conducting infinite cylinder (c) with and (d) without printed Jerusalem crosses cloak ($a_c = 1.25a$). A TM-polarized plane wave illumination is from the $+x$-direction, with the electric field polarized parallel to the z-axis.

4. Conclusion

We have presented the analysis and design of cylindrical mantle cloaks formed by slotted (printed) frequency selective surfaces, with explicit design formulas, in order to dramatically suppress the scattering from dielectric (conducting) objects. It has been shown that with a simple conformal mantle cloak, the total scattering can be significantly suppressed and available analytical formulas for these planar metasurfaces with periodic, sub-wavelength inclusions may be suitably used to tailor the required surface reactance. Our analytical results have been successfully validated using full-wave numerical simulations.

References

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