Experimental determination of effective parameters in a layered metamaterial

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Abstract

Using millimeter wave spectroscopy we extract the effective parameters of a layered metamaterial made from split ring resonators. In agreement with recent theoretical considerations the effective model for a bulk metamaterial strongly contradicts the experimental spectra. On the contrary, the description within the concept of a layered metamaterial well reproduce the transmittance and reflectance data. We demonstrate that the thickness of the metamaterial layer is not a well-defined parameter of the model. Instead, the product of the layer thickness and the dielectric susceptibility can be used to account for the experiments.

1. Introduction

An important recent discussion in the field of metamaterials is devoted to the homogenization of their effective electrodynamic properties [1, 2] such as dielectric permittivity (ε) or magnetic permeability (μ). Especially close to resonances the effective optical parameters of metamaterials often lead to physically incorrect description of the electrodynamic response. In order to be still able to control and predict the electrodynamic properties of metamaterials various concepts have been suggested, e.g. Refs. [1, 2, 3].

In this work we utilize the concept of a metafilm to describe the experimental electrodynamic response using effective values for ε and μ. To do this we use a two-layer model and the classical Fresnel equations for the complex transmission and reflection coefficients. We show that the effective parameters can in fact be used to reproduce the electrodynamic response of the metamaterial as long as the thickness of the active layer is also considered. The experimental data are supported by simulation results. With the formalism presented in this work the full experimental characterization of the metafilm can be achieved including transmittance and reflectance in the millimeter wave range.

2. Experiment

In present experiments a single layer of split-ring resonators (SRR) was used.Three samples with varying gap sizes were produced by chemical etching of copper-laminated board using an optical mask. The size of the rings in all samples is $0.4{\times}0.4\,\mathrm{mm}^2$ with a gap width of 0.15 mm, 0.25 mm and 0.3 mm and a copper thickness of 20 μm. The lattice constant of the metamaterial is 0.9 mm for all samples. As a dielectric substrate the woven glass with thickness of d_{sub}=0.48 mm has been used ($n_{sub} = 1.90{+}0.07i$).

The experiments were carried out in a quasi-optical Mach-Zehnder interferometer arrangement in the millimeter wavelength regime [4] which allows the measurement of both the intensity and the phase shift of the transmitted and reflected radiation within controlled polarization geometries.

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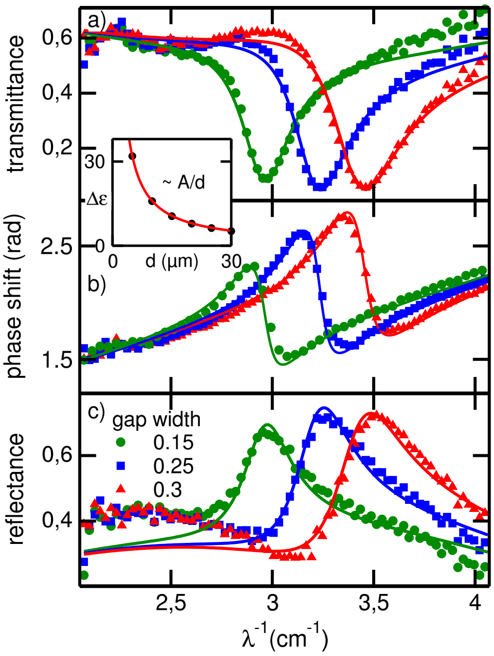

Fig. 1: Experimental millimeter wave transmittance (a), phase shift (b) and reflectance (c) spectra of SRR structures with different gap sizes. Gap width is given in millimeter. Symbols denote the experimental data, while solid lines show a fit using a two layer model. Inset in a shows the thickness dependence of the resonance strength for the sample with the gap width $g = 0.25$ mm.

In present experiments the electrical field of the radiation is chosen to be parallel to the gap of the rings, so the rings are excited electrically. Therefore the response of the rings is described by the dielectric permittivity, while the permeability can be taken as $\mu = 1$ in the geometry with electric excitation. To describe the resonance in a split ring metamaterial a Lorentz formula has been used.

3. Discussion

In a first step we try to fit the experimental data using a simple single layer model. To obtain the properties of the homogenized layer the frequency independent permittivity of the substrate is added. The Fresnel formulas for the complex transmission and reflection coefficients are then used to fit the experimental transmittance and phase shift. While this approach gives a fairly good description for the transmission and phase shift data, it fails completely in description of the reflectance data (not shown).

The situation changes drastically if a two-layer model is used to describe the metamaterial. The first layer is taken as an "effective" layer describing the response of the metafilm by a single Lorentzian and the second layer represent the properties of the substrate. The result of this approach is shown in Fig. 1. Here the solid lines show the fit with the two-layer model, while symbols show the experimental data. As can be clearly seen in Fig. 1 the transmittance, phase shift and reflectance data are almost perfectly reproduced using the two-layer model.

As a next step the measurement of the transmittance and reflectance spectra of the metamaterial (Fig. 1a,c) allow us to extract the complex values of the dielectric permittivity by numerically inverting the Fresnel equations for the amplitudes only (T,R) approach using either bulk or multi-layer concepts.

The extracted permittivity spectra within a bulk approach are shown in Fig. 2a,c. The imaginary parts of the permittivity seem to be quite reasonable and resemble a Lorentz form. On the contrary, the real part of the permittivity seem to be unphysical in the bulk model. This especially concerns the steps in the data at 2.6 cm^{-1} and 3.7 cm^{-1}, which seem to be due to Fabry-Pérot problem for the bulk concept [5]. In addition, the absolute values of the permittivity are far too large and would not agree with the measured phase shift of the transmittance (Fig. 1b).

The results of the two-layer (T,R) approach are shown Fig. 2a,c. The layered model closely coincides with a Lorentz like behavior, which supports our approach based on a single Lorentzian as discussed above. The deviations of the two-layer ε_1 from the exact Lorentz-shape at low frequencies as well as the small negative values of ε_2 could be explained by diffraction effects which could also be seen in the reflectance data (Fig. 1 c).

Since the metafilm layer is not infinitely thin, the question about the correct choice of the metafilm thickness d_{SRR} remains open. Depending on the choice of the "effective" thickness of the layer one gets different results for the permittivity. This can be seen in the inset of Fig. 1a. Here the strength of the resonance is shown in dependence of the thickness of the metafilm layer (symbols) and, as expected, a

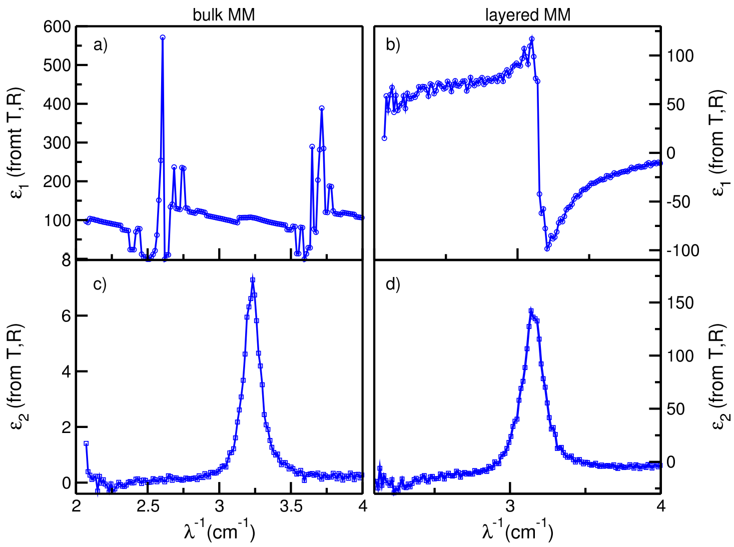

Fig. 2: Complex dielectric permittivity extracted from experimental transmittance and reflectance data using the single layer model (a,c) and the two layer model (b,d). The results are shown for the sample with the gap width $g = 0.25$ mm.

strong dependence is observed. The other free parameters, like position and width of the resonance are nearly independent of the choice of d_{SRR}. On a closer inspection, the development of the resonance strength with the layer thickness shows a nice inverse proportionality, i.e. $\Delta\varepsilon = A/d_{SRR}$, with A being a constant. This behavior is demonstrated in the insets of Fig. 1a by solid lines. This means that for a given metafilm the permittivity alone may be not a good parameter for the description, but the product $\chi \cdot d_{SRR}$ indeed is. This product remains constant in a large range of parameters and, therefore, could be used to characterize the structure.

4. Conclusion

In conclusion, we presented millimeter wave transmittance and reflectance spectra of a split ring metamaterial on a dielectric substrate. The concept of bulk averaged permittivity is clearly broken for our metamaterial samples. We demonstrate that a simple two-layer model with effective permittivity can describe the response of a metafilm quite accurately. Effective two dimensional permittivity defined as a product $\chi \cdot d_{SRR}$ becomes a new parameter governing the electrodynamics of the split rings metamaterial.

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