Losses from lossless building blocks?

A. Sihvola, H. Wallén, and H. Kettunen

Department of Radio Science and Engineering
Aalto University School of Electrical Engineering
P.O. Box 13000, FI–00076, Finland
Fax: + 358 9 47022152
email: ari.sihvola@aalto.fi, henrik.wallen@aalto.fi, henrik.kettunen@aalto.fi

Abstract

In this paper the focus is in the generation of dielectric losses from lossless or extremely low-loss component materials by a suitable mixing process. In ordinary mixing, the macroscopic continuum "inherits" the properties of its constituents. Here we show by several examples, that it is possible to break this rule. The dissipative character of dielectric materials can be drastically changed by mixing.

1 Introduction

In metamaterials engineering research, it is not uncommon that the designed structure displays unexpected properties. In fact, it is the very objective in metamaterials research to fabricate media that exhibit emergent characteristics which are not present in the constituent materials [1].

Mixing of materials with different dielectric and/or magnetic properties is a phenomenon where very small changes in the constitution of the structure may cause strong variation in the macrosopic behavior. Even in the case where the components of the mixture obey linear constitutive relations, the sensitivity of the global response on the base properties make the effective response very non-linear with respect to structural variation. Percolation behavior [2] is an example of such non-linearity.

In this presentation we focus on one potentially emergent property in dielectric homogenization: losses. Is it possible that the mixing process could give birth to dissipative character mixtures of lossless or low-loss building blocks?

2 Homogenization principles

Among the numerous mixing formulas in the electromagnetics literature, the Maxwell Garnett [3] and Bruggeman [4] homogenization principles are perhaps the most commonly used. In the fundamental form, they deal with a two-component geometry with spherical particles, and lead to formulas from which the effective permittivity of the mixture ε_{eff} is easily calculated as a function of the component permittivities and their fractional volumes.

The Maxwell Garnett and Bruggeman predictions for the effective permittivity are different in general. For mixtures made of components that do not contrast much in their dielectric response, the two predictions for ε_{eff} are fairly close to each other. However, for mixtures with strongly different permittivities, either in their real or imaginary parts, the two formulas start to deviate from each other when the mixing ratio increases.

In the case of mixtures with random structure, it is natural that there is no single and unique formula for the effective permittivity. With the same permittivity contrast and fractional volume ratio, two samples have different permittivities. Indeed, another way to look at basic mixing rules is to treat them as bounds between which the permittivity of any sample with the same structural parameters has to fall [5].

ISBN 978-952-67611-2-1

However, a much more fundamental difference between the Maxwell Garnett and Bruggeman models appears when they are applied to plasmonic mixtures. In such mixtures, the ratio between the permittivities of the two dielectric components is a negative number. In this case the Maxwell Garnett prediction leads to singularities, in other words ε_{eff} becomes infinitely large for certain combinations of the permittivity ratio and volume fraction.

In contrast, Bruggeman formula does not lead to infinities but rather its prediction is a *complex-valued* ε_{eff}! This happens even when both component permittivies are real. A non-zero imaginary part in the permittivity entails dissipation, and hence it means that a lossless whole can be built from lossless components.

Both predictions (infinities and imaginary permittivities) are intuitively difficult to accept. However, it seems that the metamaterials literature is more tolerant to the singularly behaving effective permittivity than accepting the emergence of losses. Also, numerical results [6] have given confirmation for the plasmonic Maxwell Garnett prediction of real-valued ε_{eff} which may attain extremely strong amplitudes, at least in the case of ordered mixtures.

On the other hand, in the positive-permittivity region, the Bruggeman formula is not the optimum one for regularly latticed structures, but rather it has been shown better suited for amorphous mixtures. In addition, there are cases where also the Maxwell Garnett approach with lossles components leads to complex ε_{eff}, to be discussed in the next section. Hence it may be reasonable to give a second thought on the possibility of emergent losses in certain type of metamaterials.

3 Complex responses for real-parameter inclusions

The following examples show the electric response of a lossless structure to become complex-valued.

RU sphere

The electrostatic response of a small particle is contained in its polarizability α. In the normalized form, this quantity, for an isotropic sphere reads $\alpha = 3(\varepsilon - 1)/(\varepsilon + 2)$ where ε is the relative permittivity of the sphere. For an anisotropic sphere with a constant Cartesian dyadic permittivity $\overline{\overline{\varepsilon}}$, the polarizability also becomes a dyadic quantity: $\overline{\overline{\alpha}} = 3(\overline{\overline{\varepsilon}} - \overline{\overline{I}}) \cdot (\overline{\overline{\varepsilon}} + 2\overline{\overline{I}})^{-1}$, where $\overline{\overline{I}}$ is the unit dyadic.

Another interesting type of anisotropy in spherical structures is a radial anisotropy. A radially uniaxial (RU) sphere has two different permittivity components, one in the radial ε_{r}, one in tangential ε_{t} directions with respect to the form of the sphere. The macrosopic response of such a sphere is isotropic and it turns out [7] that its polarizability is a scalar

$$\alpha = 3\frac{\sqrt{\varepsilon_{\text{r}}^2 + 8\varepsilon_{\text{r}}\varepsilon_{\text{t}}} - \varepsilon_{\text{r}} - 2}{\sqrt{\varepsilon_{\text{r}}^2 + 8\varepsilon_{\text{r}}\varepsilon_{\text{t}}} - \varepsilon_{\text{r}} + 4} \tag{1}$$

Due to the square root in this expression, it is possible that the polarizability becomes complex-valued. This happens when $-8 < \varepsilon_{\text{r}}/\varepsilon_{\text{t}} < 0$. The damping character of lossless RU spheres in the context of Mie scattering has been pointed out in [8]. See also [9, p. 239] for discussion on the branch cut.

Semi-disk

Another example of a complex-valued polarizability in the lossless-permittivity limit has been presented by Pitkonen [10]: a semi-disk. This is a two-dimensional object which in three dimension corresponds to a cylinder with semicircular cross section, excited by a transversal field.

The response of a semi-disk with isotropic permittivity ε depends on the direction of the exciting field in the two-dimensional plane, and hence it has to be characterized by two polarizability components. For negative values of the permittivity of such an object, both of these polarizability components of the semi-disk become complex-valued in the region $-3 < \varepsilon < -1/3$ [10].

Air bubbles in indefinite material

The Maxwell Garnett model can be generalized for mixtures where the inclusions or the host material is anisotropic. An example of a mixture where isotropic free-space spherical inclusions occupy a given volume fraction in uniaxially anisotropic host material has been treated in [11]. A very interesting case is when the anisotropy is indefinite, in other words, the two permittivity components have different signs. Even if these two components are purely real, the effective permittivity attains an imaginary part.

Square lattice

An interesting exact solution for the effective permittivity of a two-dimensional structure has been given in [12]. There the geometry is such where square-shaped inclusions with relative permittivity ε occupy a volume fraction $1/4$ of the area in a free-space regular square lattice. The resulting effective permittivity reads

$$\varepsilon_{\text{eff}} = \sqrt{\frac{1+3\varepsilon}{\varepsilon+3}} \tag{2}$$

It is clear from (2) that for permittivity values $-3 < \varepsilon < -1/3$, the effective permittivity ε_{eff} is complex, in fact pure imaginary. The validity of (2) has been confirmed in [13], among other analysis, by first taking a small imaginary part into ε and then letting this imaginary part go to zero.

4 Conclusion

It is rather extraordinary that a mixing process should give rise to dissipative response when the building blocks of the mixture are lossless. Media obeying such behavior can truly be considered to fall into the class of metamaterials. The above examples, however, showed that mathematically, the effective permittivity becomes complex-valued for certain choices of real-valued and negative parameters for the inclusion permittivity or components of anisotropic permittivity. The cases where the polarizability response of a single particle becomes complex correspond to a similar phenomenon: mixtures with such scatterers have complex effective permittivity. The lossless treatment is of course an idealization: in practice, losses are inevitable in the component materials. The conclusion is, nevertheless, that with extremely small losses to begin with, considerable macroscopic losses can be generated. The relative increase of the loss factor can therefore, by a clever mixing process, be designed to be arbitrarily large.

References

[1] A. Sihvola, "Metamaterials in electromagnetics," *Metamaterials*, vol. 1, no. 1, pp. 2–11, 2007.

[2] R. Landauer, "Electrical conductivity in inhomogeneous media," *AIP Conference Proceedings*, vol. 40, no. 1, pp. 2–45, 1978. [Online]. Available: `http://link.aip.org/link/?APC/40/2/1`

[3] J. C. Maxwell Garnett, "Colours in metal glasses and metal films," *Transactions of the Royal Society (London)*, vol. 203, pp. 385–420, 1904.

[4] D. A. G. Bruggeman, "Berechnung verschiedener physikalischer Konstanten von heterogenen Substanzen. I. Dielektrizitätskonstanten und Leitfähigkeiten der Mischkörper aus isotropen Substanzen," *Annalen der Physik*, vol. 24, no. 7-8, pp. 636–679, 1935.

[5] Z. Hashin and S. Shtrikman, "A variational approach to the theory of the effective magnetic permeability of multiphase materials," *Journal of Applied Physics*, vol. 33, no. 10, pp. 3125–3131, 1962.

[6] H. Wallén, H. Kettunen, and A. Sihvola, "Mixing formulas and plasmonic composites," in *Metamaterials and plasmonics: fundamentals, modelling, applications*, ser. NATO Science for Peace and Security Series, B – Physics and Biophysics, S. Zouhdi, A. Sihvola, and A. Vinogradov, Eds., Springer, Dordrecht, 2009, pp. 91–102.

[7] T. Rimpiläinen, H. Wallén, H. Kettunen, and A. Sihvola, "Electrical response of systropic sphere," submitted.

[8] C.-W. Qiu and B. Luk'yanchuk, "Peculiarities in light scattering by spherical particles with radial anisotropy," *Journal of the Optical Society of America A*, vol. 25, no. 7, pp. 1623–1628, July 2008.

[9] G. W. Milton, *The Theory of Composites*. Cambridge University Press, 2002.

[10] M. Pitkonen, "A closed-form solution for the polarizability of a dielectric double half-cylinder," *Journal of Electromagnetic Waves and Applications*, vol. 24, no. 8-9, pp. 1267–1277, 2010.

[11] A. Sihvola, "Metamaterials and depolarization factors," *Progress in Electromagnetics Research*, vol. 51, pp. 65–82, 2005.

[12] Y. V. Obnosov, "Periodic heterogeneous structures: New explicit solutions and effective characteristics of refraction of an imposed field," *SIAM Journal on Applied Mathematics*, vol. 59, no. 4, pp. 1267–1287, 1999. [Online]. Available: `http://link.aip.org/link/?SMM/59/1267/1`

[13] J. Helsing, R. C. McPhedran, and G. W. Milton, "Spectral super-resolution in metamaterial composites," *New Journal of Physics*, vol. 13, no. 11, p. 115005, 2011.

