Low-loss tunable metamaterials using superconducting thin-film circuits with Josephson junctions

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Abstract

We report on experiments with superconducting Josephson metamaterials. In such a material, conventional split ring resonators are replaced by superconducting loops that are interrupted by Josephson junctions, so called rf-SQUIDs. Like the split ring resonators, they can be seen as LC-resonators that couple to the magnetic component of the incoming wave. The advantage of superconducting thin-film metamaterials is that due to the tunable intrinsic inductance of the Josephson junction, the resonance frequency of the rf-SQUID can be changed by applying an external dc magnetic field. We present experimental results that show the tunability of the resonance frequency of these devices.

One of the main limitations of many metamaterial designs is the restriction of their usability to a narrow frequency band. By using resonant elements in order to achieve the desired effects (such as a negative μ_r) the “meta-atoms” are usually made for a fixed operation frequency range by design. To circumvent this limitation, several approaches have been developed [1], many of which are based on the idea of introducing a nonlinear element into the resonant circuit, thus making it tunable [2,3]. Superconductors in general and Josephson junctions in particular are ideal constituents of tunable meta-atoms due to the strong dependence of their inductance on external parameters such as temperature and magnetic field. The tunability by varying temperature has already been demonstrated [4]. In this work, we now move one step further and experimentally demonstrate tunability via magnetic field by replacing the SRR by an rf-SQUID (Superconducting QUantum Interference Device). This approach was previously suggested in the theoretical work by N. Lazarides *et al.* [5].

Superconductivity and the physics of Josephson junctions offer two main advantages to the field of microwave metamaterials. First, losses in the material are drastically reduced. Second, the intrinsic inductance of the Josephson junction can be tuned by changing the dc current through the junction. To understand this Josephson inductance L_j, one has to look at the fundamental equations that govern the voltage and current over such an element, namely the Josephson equations:

$$I = I_c \sin\varphi \qquad \text{(1}^{\text{st}}\text{ Josephson equation)} \tag{1}$$

$$V = \frac{\Phi_0}{2\pi}\dot{\varphi} \qquad \text{(2}^{\text{nd}}\text{ Josephson equation)} \tag{2}$$

Here $\Phi_0 = \frac{h}{2e}$ is the flux quantum, I_c the critical current of the junction, and φ the phase difference of the superconducting wave function over the junction. For a small variation of the phase difference $\varphi = \bar{\varphi} + \delta\bar{\varphi}$, current and voltage can be related to each other by using (1) and (2) [6]:

$$V = \frac{\Phi_0}{2\pi I_c \cos(\bar{\varphi})}\frac{d}{dt}I \equiv L_j(\bar{\varphi})\frac{d}{dt}I \tag{3}$$

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This equation takes the characteristic form of the equation for an inductive element.

The circuit consisting of such a Josephson junction placed into a superconducting loop (cf. Fig 1a) is called an rf-SQUID and the magnetic flux quantization leads to the relation

$$\varphi + \frac{2\pi L_{geo}}{\Phi_0} I = \frac{2\pi}{\Phi_0} \Phi_{\text{ext}} \tag{4}$$

An external magnetic flux $\Phi_{\text{ext}} = \Phi_{\text{e0}} + \Phi_{\text{ea}} \cos(\omega t)$ induces a circulating current in the loop. According to Eq. 1, this changes the phase difference over the junction. For the case $\Phi_{\text{ea}} \ll \Phi_{\text{e0}}$ and $|\Phi_{\text{e0}}|$ reasonably far away from $(2n-1) \cdot \frac{\Phi_0}{2}$ with $= 1,2,3, \ldots$, the Josephson inductance depends only on the dc component of the flux Φ_{e0}. The equivalent small signal circuit model for such an rf-SQUID is shown in Fig. 1c. It resembles a classical LC resonator with a tunable inductance.

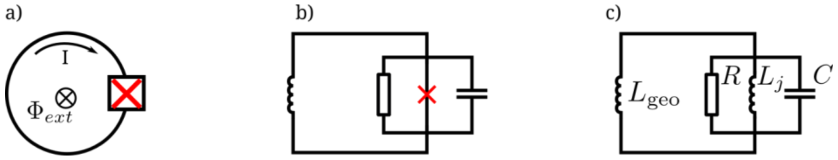

Fig. 1: (a) A superconducting loop interrupted by a Josephson junction is decomposed into a (b) network model using the RCSJ-model [7]. The red cross indicates an element of which voltage and current are given by the Josephson equations (1), (2). (c) Small signal network model of the rf-SQUID. L_{geo} presents the geometric inductance of the superconducting loop. L_j is the Josephson inductance (Eq. 3). R and C are the normal resistance and capacitance of the Josephson junction, respectively. In our sample, the junction is shunted by a large additional capacitance to reduce the resonance frequency, which is included in C.

The additional kinetic inductance of the superconducting ring has been omitted since in the present experiment, it has been designed to be much smaller than the geometric inductance at the operating temperature. However, close to the critical temperature of the superconductor the kinetic inductance will become significant. Due to this and the temperature dependence of the critical current of the junction, the rf-SQUID also shows an intrinsic tunability by temperature which will not be detailed here.

Although the approximation of the Josephson junction as a tunable inductor works well in the mentioned limit, a complete description includes the Josephson nonlinearity and loss terms. It can be derived by applying Kirchhoff's laws and (1), (2) and (4) to the circuit depicted in Fig. 1b and takes the form of a second order nonlinear differential equation:

$$\varphi + \beta_L \left[\sin(\varphi) + \frac{1}{\omega_c} \dot{\varphi} + \frac{1}{\omega_p^2} \ddot{\varphi} \right] = \varphi_{e0} + \varphi_{ea} \cos(\omega t) \tag{5}$$

Here $\varphi_{e0} = \frac{2\pi}{\Phi_0} \Phi_{\text{e0}}$, $\varphi_{ea} = \frac{2\pi}{\Phi_0} \Phi_{\text{ea}}$, $\beta_l = \frac{2\pi\, L_{geo}\, I_c}{\Phi_0}$, $\omega_c = \frac{2\, I_c\, R\, e}{\hbar}$, and $\omega_p^2 = \frac{2\, e\, I_c}{C\, \hbar}$. By solving this equation numerically, one can extract the effective magnetic susceptibility χ_{mag}.

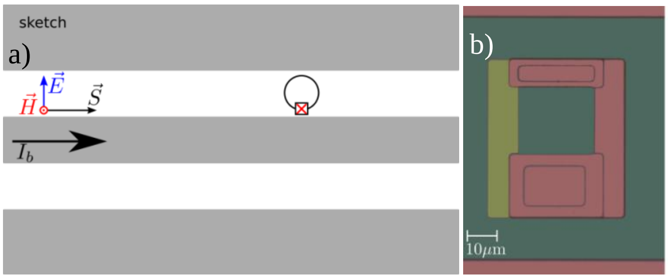

Fig. 2: (a) Sketch of a single rf-SQUID embedded into a coplanar transmission line. Only magnetic coupling is assumed. (b) Optical microscope image of a single rf-SQUID in the gap of the coplanar waveguide.

To verify the properties of an individual SQUID, we have designed and fabricated a circuit with the SQUID placed inside the gap of a coplanar waveguide. The SQUID loop is perpendicular to the magnetic field induced by the current I_b flowing along the central conductor of the coplanar waveguide, as shown in Fig. 2a. The arrows denoted by $\vec{E}$, $\vec{H}$ and $\vec{S}$ point in the direction of the electric and magnetic field components and the Poynting vector of the incoming wave, respectively. By applying an external dc magnetic field the resonance frequency of SQUID meta-atoms is tuned in situ.

The SQUID used in the presented measurement is made from Nb using a $Nb - AlO_x - Nb$ tunnel junction. The geometric inductance L_{geo} = 63.5pH is comparable to the Josephson inductance L_j = 74pH at zero external flux. By shunting the Josephson junction with a large capacitor C_{shunt} = 1.7pF we estimated the resonance frequency to be tunable between about 5GHz and 21GHz for an infinitesimally small φ_{ea} . The measurements were performed using a cryogenic amplifier and a vector network analyzer while the sample was immersed in liquid ^4He at a temperature of 4.2K.

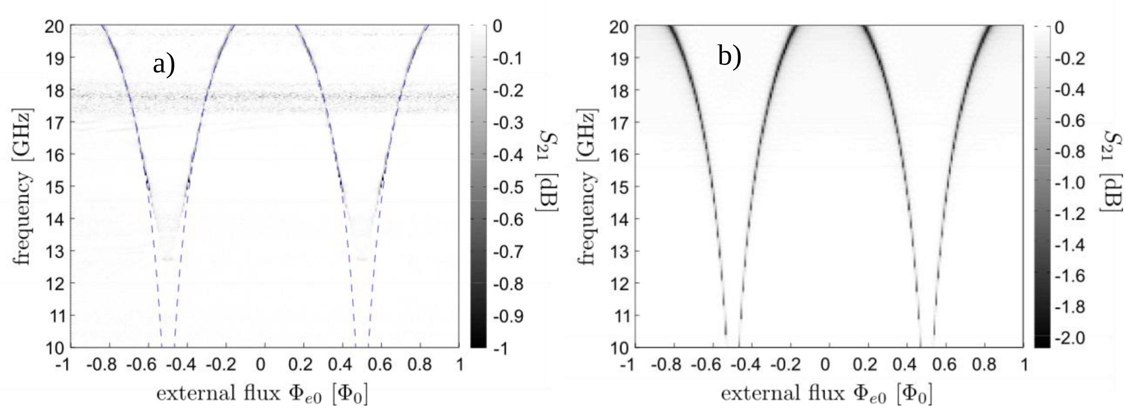

Fig. 3: (a) Measured transmission with $P \approx -85 dBm$ at the sample. The blue dashed line represents the analytical solution extracted from the small signal model depicted in Fig. 1c. (b) Simulated transmission (solving Eq. 5) of a single SQUID in a transmission line.

From the low-power transmission data of Fig. 3a one can immediately see the expected frequency tunability of the resonance dip corresponding to the dark color in the plot. The blue dashed line represents a fit of the analytical small signal model to the measured data. As discussed earlier, the model should yield reasonably good results only for flux bias values far away from $\Phi_{e0} = (2n + 1)\frac{\Phi_0}{2}$. The obtained fit parameters for L, C and I_c are within 6% of the expected values. The results from the fit were then introduced into the numerical simulation. The result of this can be seen in Fig. 3b and is in good agreement with the measured data.

In conclusion, we have experimentally and theoretically investigated a possible new type of magnetically coupled, tunable meta-atom. The rf-SQUID is a very promising field-tunable alternative to the common split ring resonator. Further studies on SQUID properties at higher powers, as well as experiments on 1D-arrays of such elements are ongoing.

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