Negative-index metamaterials composed of three-concentric dielectric or magnetodielectric spheres

B. Tomasic[1], K. Kim[2] and N. Herscovici [3]

Electromagnetics Technology Division, Sensors Directorate
Air Force Research Laboratory
Wright-Patterson AFB OH 45433-7302 USA
[1]Fax: 937-255-8656; email: boris.tomasic@wpafb.af.mil
[2]Fax: 937-255-8656; email: kristopher.kim@wpafb.af.mil
[3]Fax: 937-255-8656; email: naftali.herscovici@wpafb.af.mil

Abstract

In this paper we analyze an infinite periodic planar array of multiple-layer spheres where each concentric layer is made of different electric and/or magnetic materials. Each sphere can have up to three concentric layers to facilitate the optimization of metamaterial structure with respect to performance and ease of fabrication processes. The structure is analyzed by modifying the method developed in [1] for homogeneous magnetodielectric spheres. Numerical results for the effective constitutive parameters of representative examples are given.

1. Introduction

Metamaterials composed of homogeneous magnetodielectric spheres have attracted much attention because of their almost isotropic and low-loss properties, which make them candidates for many applications, e.g., lenses of sub-wavelength resolution [2,3]. Although the loss of these metamaterials is low relative to alternative designs, such as the SRR-wire element, it is not low enough for most practical applications. In addition, these materials, which require spheres with high permeabilities and permittivities, are difficult to realize. For this reason, in [4] and later in [5], the authors proposed a more practical approach where the unit cell consists of two dielectric spheres with different radii and/or permittivities chosen so that the electric resonance of one sphere coincides with the magnetic resonance of the other sphere. This has the advantage of employing only dielectric materials which are inherently of low loss, but suffers from structural and fabrication complexity. To mitigate these weaknesses, in [6] the authors proposed a cubic periodic array of two-layered spheres, made from low-loss, high-permittivity ceramics. A "brute force" search was carried out by varying the radii of the shell and core of the layered sphere until a combination was found such that the real parts of ϵ_{eff} and μ_{eff} both become negative over the same frequency band.

In this work we have extended our theory developed in [1] for the analysis of infinite periodic arrays of homogeneous magnetodielectric spheres to spheres with multiple dielectric and/or magnetic materials in concentric layers. The spherical particles can have up to three layers, which offers greater flexibility in the design of negative-index metamaterials. As described in [1], first we calculated the scattering coefficients of the single sphere in a unit cell. Consequently, we retrieve the effective constitutive parameters ϵ_{eff} and μ_{eff} using the standard transmission line method. In the analytical process it is possible to separate the array geometry from the element geometry. As a result, we only need to modify the element part due to the change introduced by the new three-layer sphere geometry. The parametric optimizer in Matlab was implemented to search for a combination of material and geometric parameters that results in a negative-index behavior over a frequency range.

ISBN 978-952-67611-2-1

2. Analysis

A single element of an infinite 2D array is shown in Fig.1. It consists of three concentric layers with radii a_i, $i = 1, 2, 3$. Each layer is made up of material specified by relative permittivity and permeability ($\epsilon_{ri},\ \mu_{ri}$). The analysis for an infinite planar array of uniform magnetodielectric spheres developed in [1] directly applies here except for the part that evaluates scattering coefficients of the TE^r and TM^r modes, a^r_{nm}, b^r_{nm}, respectively, with (n, m) being spherical harmonics indices. In this case the total field inside the sphere $\mathbf{E}^i$ and the scattered field $\mathbf{E}^r$ are represented in terms of vector spherical harmonics $\mathbf{M}_{nm}$ and $\mathbf{N}_{nm}$ as follows:

$$\mathbf{E}^1(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a^1_{nm}\mathbf{M}_{nm}(k_1\mathbf{r}) + b^1_{nm}\mathbf{N}_{nm}(k_1\mathbf{r})],\ 0 \le r \le a_1 \tag{1}$$

$$\begin{aligned}\mathbf{E}^2(\mathbf{r}) &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a^{2+}_{nm}\mathbf{M}^+_{nm}(k_2\mathbf{r}) + b^{2+}_{nm}\mathbf{N}^+_{nm}(k_2\mathbf{r})] \\ &+ \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a^{2-}_{nm}\mathbf{M}^-_{nm}(k_2\mathbf{r}) + b^{2-}_{nm}\mathbf{N}^-_{nm}(k_2\mathbf{r})],\ a_1 \le r \le a_2\end{aligned} \tag{2}$$

$$\begin{aligned}\mathbf{E}^3(\mathbf{r}) &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a^{3+}_{nm}\mathbf{M}^+_{nm}(k_3\mathbf{r}) + b^{3+}_{nm}\mathbf{N}^+_{nm}(k_3\mathbf{r})] \\ &+ \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a^{3-}_{nm}\mathbf{M}^-_{nm}(k_3\mathbf{r}) + b^{3-}_{nm}\mathbf{N}^-_{nm}(k_3\mathbf{r})],\ a_2 \le r \le a_3\end{aligned} \tag{3}$$

$$\mathbf{E}^r(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a^r_{nm}\mathbf{M}^+_{nm}(k_0\mathbf{r}) + b^r_{nm}\mathbf{N}^+_{nm}(k_0\mathbf{r})],\ r \ge a_3. \tag{4}$$

Here, with $exp(-i\omega t)$ time dependence, the radial dependence of $\mathbf{M}_{nm}(x)$, $\mathbf{N}_{nm}(x)$ is given by spherical bessel functions $j_n(x)$, while for $\mathbf{M}^+_{nm}(x)$, $\mathbf{N}^+_{nm}(x)$, and $\mathbf{M}^-_{nm}(x)$, $\mathbf{N}^-_{nm}(x)$ the radial dependence is given by the spherical Hankel functions $h^{(1)}_n(x)$and $h^{(2)}_n(x)$ representing the spherical waves propagating in positive and negative radial directions, respectively.

To solve for the expansion coefficients a_{nm}, b_{nm} in (1)-(4), we apply the following boundary conditions: $\hat{\mathbf{r}} \times \mathbf{E}^1(a_1) = \hat{\mathbf{r}} \times \mathbf{E}^2(a_1)$, $\hat{\mathbf{r}} \times \mathbf{E}^2(a_2) = \hat{\mathbf{r}} \times \mathbf{E}^3(a_2)$, and $\hat{\mathbf{r}} \times \mathbf{E}^3(a_3) = \hat{\mathbf{r}} \times \mathbf{E}^r(a_3) + \hat{\mathbf{r}} \times \mathbf{E}^{inc}(a_3)$. The total incident field on the reference element $(p, q) = (0, 0)$ is $\mathbf{E}^{inc}(a_3) = \mathbf{E}^{inc}_0(a_3) + \sum_{pq\neq(0,0)}\mathbf{E}^r_{pq}(a_3)$ where the first term on the right hand side represents the plane wave incident on the reference element while the second term is the incident field on the reference element scattered from other array elements. Similar expressions and boundary conditions hold for the $\mathbf{H}$ field. These boundary conditions yield a system of inhomogeneous equations for the expansion coefficients in (1)-(4).

3. Results

The TE^r mode (magnetic) resonance occurs when the denominator of a^r_{nm} vanishes, and similarly the TM^r mode (electric) resonance occurs when the denominator of b^r_{nm} vanishes. Magnetic resonances are dictated by the zeros of the nth order spherical Bessel functions while the electric resonances are governed by the zeros of the derivative of the spherical Bessel function of order n. For a sphere made up of homogeneous magnetodielectric material, the first electric resonance is always below the first magnetic resonance. When the magnetic and electric resonances occur at the same frequency, the doubly negative zone, where both effective constitutive parameters ϵ_{eff} and μ_{eff} are negative, lies in the region slightly beyond the resonance line. The main objective of this work is to increase the number of sphere parameters which give us more flexibility in search for geometrical and material configurations that will align magnetic and electric resonances. Dielectric only materials are desirable since they produce low-loss, negative-index metamaterials. In the presentation it will be shown that it is possible to produce electric and magnetic responses at the same frequency using dielectric materials only. As a simple example, Figure 2 shows a plot of the effective ϵ_r and μ_r vs. frequency for the metamaterial composed of a single planar layer of spheres with a copper core and two dielectric shells. The parameters of the sphere element are also provided in Figure 2. We observe that the index of refraction is negative in the region 2.9 to 3.2 GHz, thus yielding the bandwidth of approximately 10%.

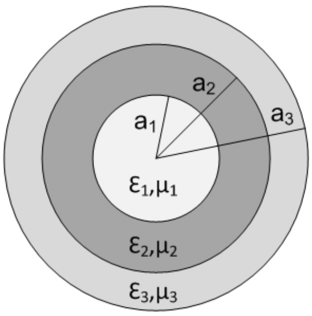

Fig. 1: A three-layered sphere

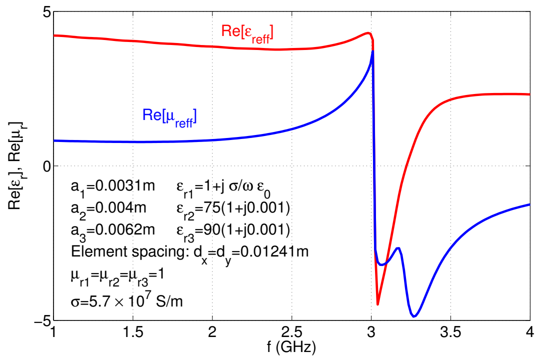

Fig. 2: Effective ϵ_r and μ_r for planar array of spheres with a copper core and two dielectric shells, n=1 spherical harmonic considered

4. Conclusion

We have developed a method for determining the effective constitutive parameters ϵ_{eff} and μ_{eff} of a planar array of multi-layer concentric spheres, thus offering greater flexibility to search for negative index configurations as a function of frequency. Higher order spherical multipoles have been included in the analysis. The method is demonstrated in an example of single layer of spheres with a copper core and two dielectric shells, exhibiting a negative index of refraction over 10% frequency bandwidth.

References

[1] B. Tomasic, K. Kim and N. Herscovici, Analysis of metamaterial slab of magnetodielectric spheres, *Fifth International Congress on Advanced Electromagnetic Materials in Microwaves and Optics*, October 2011, Barcelona, Spain.

[2] C.L. Holloway, E.F. Kuester, J. Baker-Jarvis, and P. Kabos, A doubly negative (DNG) composite medium composed of magneto-dielectric spherical particles embedded in a matrix, *IEEE Trans. Antennas Propag.*, vol. 51, pp. 2596-2603, 2003.

[3] R.A. Shore and A.D. Yaghjian, Traveling waves on two- and three-dimensional periodic arrays of lossless scatterers, *Radio Science*, vol. 42, 2007.

[4] O.G. Vendik and M.S. Gashinova, Artificial double negative (DNG) media composed of two different dielectric spheres lattices embedded in a dielectric matrix, *34th Eur. Microw. Conf.*, pp. 1209-1212, 2004.

[5] S. Ghadarghadr and H. Mossallaei, Dispersion diagram characteristics of periodic array of dielectric and magnetic materials based spheres, *IEEE Trans. on Antennas and Prop.*, vol. 57, no. 1, January 2009.

[6] E.F. Kuester, N. Memic, S. Shen, A. Scher, S. Kim, K. Kumley, and H. Loui, A negative refractive index metamaterial based on a cubic array of layered nonmagnetic spherical particles, *Progress in Electromagnetics*, Research B, vol. 33, pp. 175-202, 2011.