Advantages of metamaterials based on double-stranded DNA-like helices

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Abstract

Here we consider three optimal helices: double-stranded (ds) DNA-like helix of one half-turn, optimal single-stranded (ss) helices of one- and two-turns, and compare the ratios of polarizabilities for them. Metamaterials with equal permittivity and permeability can be created on the basis of such optimal helices. It is shown that ds-DNA-like helix has highest polarizabilities among other optimal helices. This advantage can be used for creation metamaterials with negative refraction of electromagnetic waves.

1. Introduction

In previous works [1-3] we have presented the analytical relationships between the dielectric, magnetic, and chiral (magnetoelectric) polarizabilities for short (of several pitches) metallic helices. It was shown that there is an “optimal” ratio between the radius and helix pitch, such that all three polarizabilities are equal at a certain frequency (this ratio was introduced previously in [1–3] for the helices used as converters of polarization). Any helix is characterized by dielectric, magnetic, and chiral polarizabilities in the field of monochromatic $\exp(j\omega t)$ electromagnetic wave, therefore the next coupling equations can be written

$$\vec{p} = \varepsilon_0 \alpha_{ee} \vec{E} - j\sqrt{\varepsilon_0 \mu_0}\,\alpha_{em} \vec{H}\ ,\ \vec{m} = \alpha_{mm} \vec{H} + j\sqrt{\varepsilon_0 / \mu_0}\,\alpha_{me} \vec{E} \qquad (1)$$

Here, α_{ee} and α_{mm} are the tensors of dielectric and magnetic polarizabilities; α_{em} and α_{me} are the pseudotensors characterizing the chiral properties of the helix; ε_0 and μ_0 are the electric and magnetic constants, respectively. The Onsager–Casimir principle of symmetry of kinetic coefficients yields the relationship [4] $\alpha_{em} = \alpha_{me}^{T}$, where *T* denotes tensor transposition. Both the electric and magnetic moments (1) refer to the same building block of helix with a certain current distribution, which is determined by the shape and size of this block. This results correlate between the electric and magnetic moments induced in the helix. The axial components of these two moments are related as [5, 6]

$$p_x = -2jm_x / \omega r^2 q \qquad (2)$$

where *x* denotes the direction along the helix axis, *r* is the helix radius, *q*= 2π/*h* (*h* is the helix pitch), and ω is the angular frequency of the current. The sign of specific twisting *q* is determined by the helix handedness: *q* > 0 for a right-hand helix. For the helices of “optimal” shapes the condition is satisfied

$$\omega |q| r^2 / c = 2 \tag{3}$$

For an isotropic medium with a low concentration of inclusions with optimal shape, we can neglect the interaction between the structure elements and define the effective parameters as $\varepsilon_r = \mu_r = 1 \pm \kappa$ (where c is the speed of light in vacuum, the upper sign corresponds to the right-handed helix).
Simultaneous use of relations (1) and (2) results in the next equations [7]

$$\alpha_{ee}^{(11)} = \alpha_{mm}^{(11)},\ \alpha_{ee}^{(11)} = \pm\alpha_{em}^{(11)} \tag{4}$$

where $\alpha^{(ik)}$ are the components of the considered tensors and pseudotensors; the plus and minus signs correspond to the right- and left-handed helices, respectively. Relations (4) show that the helices with the found optimal parameters are characterized by three equal polarizabilities for the fields directed along the helix axis: dielectric, magnetic, and chiral. The equality of all three axial polarizabilities of the optimal helices is confirmed experimentally, in particular, by the emission of a circularly polarized wave by the optimal helix in the direction normal to the helix axis [1–3, 5, 7]. Optimal helices can find wide application, e.g., for the fabrication of reflection-free coatings and metamaterials with a negative refraction of electromagnetic waves. The helices have optimum characteristics regardless of the type of activation along the helix axis (electric or magnetic), in other words for any orientation of the polarization plane of incident wave. This is one of advantages of the optimal helices over other possible elements of metamaterials, for example, those based on straight wires and split-ring resonators.

2. Dielectric, magnetic and chiral polarizabilities

Taking into account helical trajectories of the conduction electrons, the nonuniformity of the current distribution along the conductor, the skin effect, and the electric-field attenuation in metal, one can write the frequency dependence of the effective parameters of an isotropic medium

$$\varepsilon_r = 1 + \frac{1}{A\varepsilon_0}\frac{\omega_0^2 - \omega^2 - j\omega\Gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\Gamma^2},\ \mu_r = 1 + \frac{1}{A}\mu_0 B^2 \frac{\omega_0^2 - \omega^2 - j\omega\Gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\Gamma^2},\ \kappa = \frac{1}{A}\sqrt{\frac{\mu_0}{\varepsilon_0}} B \frac{\omega_0^2 - \omega^2 - j\omega\Gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\Gamma^2} \tag{5}$$

The following parameters are used in Eqs. (5):

$$\frac{1}{A} = \frac{2Ne^2\tau\sin^2\alpha}{\pi m_e},\ B = \frac{\cos^2\alpha}{\sin\alpha}\cdot\frac{c}{4N_t},\ \Gamma = \frac{\rho N_0 N_s e^2}{m_e},\ N = N_0 N_s N_h V_h,\quad V_h = \pi r_0^2 L$$

Here, V_h is the volume of the wire used to fabricate one helix, ω_0 is resonance oscillation frequency, L is the helix length, α is the pitch angle, ρ is the metal resistivity, N_0 is the volumetric concentration of conduction electrons in metal, N_h is the concentration of inclusions, N_s is the fraction of the skin layer in the helix volume, r_0 is the wire radius, N_t is number of turns in the helix, τ is coefficient of the field attenuation inside the metal, $-e$ is the electron charge, m_e is the electron mass. It is taken into account that, on the condition of the main resonance, the total helix length is approximately $\lambda_0/2$, where λ_0 is the wavelength of the electromagnetic wave in free space.

Here we consider the optimal ds-DNA-like helix of one half-turn (pitch angle 24.5 degrees) [8, 9] and optimal ss-helices of one- and two-turns (pitch angle 13.65 degrees and 7.1 degrees) (see Fig. 1) and compare the ratios of polarizabilities for them as it is shown in Tab.1. For all three helices we have the same type of resonance: the length of the helix equals the half of the wave length.

Fig. 1: From left to right: the optimal ds-DNA-like helix of one half-turn, the optimal ss-helix of one turn, and the optimal ss-helix of two-turns

Tab. 1. Expressions for the ratios of dielectric, magnetic and chiral polarizabilitiess

Ratio	$\frac{\alpha_{ee}^{1}}{\alpha_{ee}^{2}}$	$\frac{\alpha_{mm}^{1}}{\alpha_{mm}^{2}}$	$\frac{\alpha_{em}^{1}}{\alpha_{em}^{2}}, \frac{\alpha_{me}^{1}}{\alpha_{me}^{2}}$
Expression	$2\frac{\sin^2\alpha_1}{\sin^2\alpha_2}$	$2\frac{N_2^2\cos^4\alpha_1}{N_1^2\cos^4\alpha_2}$	$2\frac{N_2\cos^2\alpha_1\sin\alpha_1}{N_1\cos^2\alpha_2\sin\alpha_2}$
ds-h of half-turn/ss-h of 1-turn	6.18	6.15	6.15
ds-h of half-turn/ss-h of 2-turn	21.9	22.4	22.3

Here α_1 and α_2 are pitch angles, N_1 and N_2 are number of turns in the helix.

3. Conclusion

We have found that for the mentioned optimal helices polarizabilities α_{ee}, α_{mm} and α_{em} are equal. This significant result is possible due to the fact that helices mentioned are optimal among others with the same number of turns. One can see by comparison that the polarizability for ds-DNA-like helix is the highest among all values for optimal helices.

The advantages of metamaterials based on ds-DNA-like helices are following: for one half-turn element $\alpha_{ee}^{(11)} = \alpha_{mm}^{(11)}$, $\alpha_{ee}^{(22)} = \alpha_{ee}^{(33)} = \alpha_{mm}^{(22)} = \alpha_{mm}^{(33)} = 0$ because symmetry of double helices; $\alpha_{ee}^{(11)}$ for separate helix is higher than for helix of one turn and two turns; the chirality of metamaterials can be compensated using pairs of right and left double helical elements. These findings are of interest for engineering the new metamaterials, circular broadband polarizers [10-13], antennas of circular polarization for up to ultraviolet range, circular dichroism spectroscopy etc.

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