Calculation and analysis of the tensors of electric, magnetic and chiral polarizabilities of the helices with optimal shape

I. A. Faniayeu[1], V. S. Asadchy[1,2], I. V. Semchenko[1], S. A. Khakhomov[1]

[1]Department of Physics, Gomel State University
Sovetskaya Str. 104, 246019, Gomel, Belarus
Fax: +375232578111; email: bratya.i@mail.ru
[2]Department of Radio Science and Engineering, Aalto University
Espoo, Finland

Abstract

In this paper all components of tensors of electric, magnetic and chiral polarizabilities of one-turn and double-turn helices are calculated. The equality of axial components of above-named polarizabilities has been confirmed at the frequency close to the resonance. The comparison of frequency dispersion and anisotropic properties of the polarizabilities for both kinds of the helices is performed.

1. Introduction

Metal helices are widely used as inclusions in the metamaterials. They are characterized by electric, magnetic and chiral polarizabilities. The optimal geometric parameters of the helices were determined before. These parameters provide the equality of axial components of polarizabilities at the frequency close to the resonance. It is necessary to calculate all tensors components of the polarizabilities for determination of anisotropic properties of metamaterials. The results of calculation make it possible to choose more preferable helices for creation of metamaterials. Particularly the helices with the optimal parameters can be used for wave flow method cloaking as elements of non-scattering claddings.

2. Calculation of the polarizabilities tensors of the helices with the optimal shape

In this section determination of tensors components of the polarizabilities for a helix is performed. The helices are characterized by the optimal parameters [1]:

The one-turn helix: $N_T=1$, $\alpha=13.6^\circ$, $L=0.05$ m, $r=7.75\cdot10^{-3}$ m, $h=12\cdot10^{-3}$ m, $d=1.51\cdot10^{-3}$ m.
The double-turn helix: $N_T=2$, $\alpha=7.1^\circ$, $L=0.05$ m, $r=3.95\cdot10^{-3}$ m, $h=3.1\cdot10^{-3}$ m, $d=0.8\cdot10^{-3}$ m,

where α is the helix angle with the plane perpendicular to the helix axis, L is the length of the wire forming the helix, r is the turn radius, h is the helix pitch, and d is the wire diameter. Its behavior in an electromagnetic field can be described by the coupling equations [2]:

$$\vec{p} = \varepsilon_0 \alpha_{ee} \vec{E} - j\sqrt{\varepsilon_0 \mu_0}\, \alpha_{em} \vec{H} \qquad (1)$$

ISBN 978-952-67611-2-1

$$\vec{m} = \alpha_{mm}\vec{H} + j\sqrt{\frac{\varepsilon_0}{\mu_0}}\alpha_{me}\vec{E} \qquad (2)$$

Here α_{ee} and α_{mm} are the dielectric and magnetic polarizability tensors, α_{em} and α_{me} are the pseudo tensors describing the chiral properties of the helix, and ε_0 and μ_0 are the electric and magnetic constants. In the dipole approximation, the intensity of the electric filed of the radiated wave has the form [3]:

$$\vec{E}(\vec{R},t) = \frac{\mu_0}{4\pi R}\left(\left[\left[\ddot{\vec{p}}\ \vec{n}\right]\vec{n}\right] + \frac{1}{c}\left[\vec{n}\ \ddot{\vec{m}}\right]\right) \qquad (3)$$

where $\vec{R}$ is the radius vector passing from a helix to the observation point, R is the distance from a helix to the observation point, $\vec{n}$ is the unit vector of the wave normal, c is the velocity of light in vacuum, and points above the vectors denote time differentiation. Let us use limitations of helix symmetry and Onsager's reciprocal relations so that we should calculate components of tensors α_{ee}, α_{em}, α_{me}, α_{mm}. The one-turn and double-turn helices with the optimal shape are modeled with ANSYS HFSS (a commercial finite element method simulator). The simulation model has a single helix, made of perfect electric conductor (PEC), in free space. As the excitation, linear-polarized incident electromagnetic waves with different parameters are used. The different choose of incident wave parameters provides to calculate different components of the polarizabilities tensors. Final expressions for components of the tensors are following:

1) $E_x\neq 0$, $H_y\neq 0$ – these vectors determine polarization of incident electromagnetic wave

$$\alpha_{ee}^{11} = \frac{{}_yE_1 + {}_{-y}E_1}{A},\ \alpha_{em}^{22} = \frac{{}_zE_2 + {}_{-z}E_2}{A}j,\ \alpha_{em}^{32} = \frac{{}_yE_3 + {}_{-y}E_3}{A}j,\ \alpha_{me}^{11} = \frac{{}_{-y}E_3 - {}_yE_3}{A}j,\ \alpha_{mm}^{22} = \frac{{}_zE_1 - {}_{-z}E_1}{A} \qquad (4)$$

2) $E_y\neq 0$, $H_x\neq 0$:

$$\alpha_{ee}^{22} = \frac{{}_zE_2 + {}_{-z}E_2}{A},\ \alpha_{me}^{22} = \frac{{}_{-z}E_1 - {}_zE_1}{A}j,\ \alpha_{me}^{32} = \frac{{}_yE_1 - {}_{-y}E_1}{A}j \qquad (5)$$

3) $E_z\neq 0$, $H_x\neq 0$:

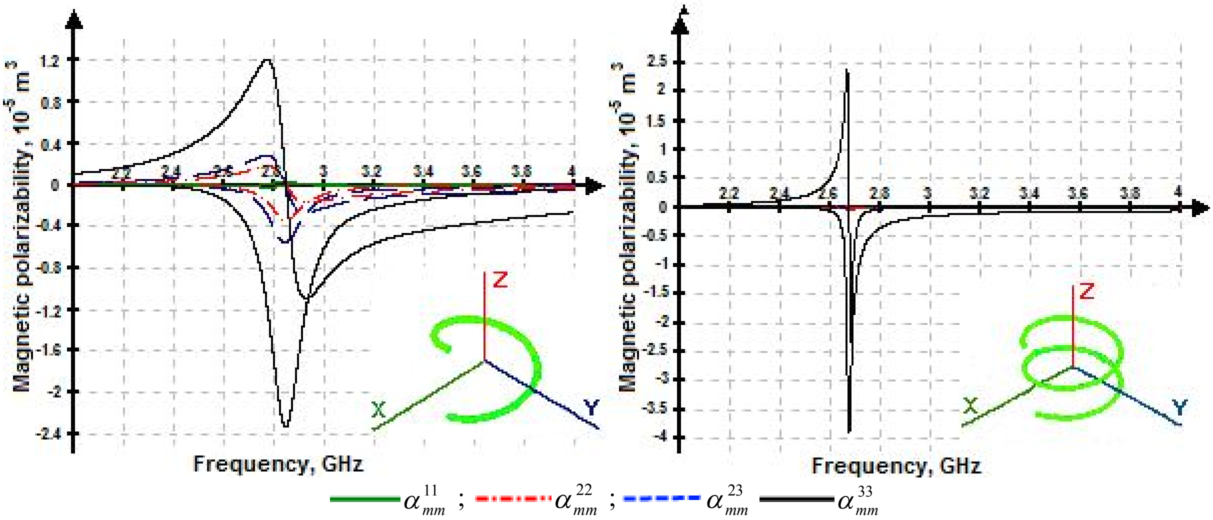
$$\begin{aligned} &\alpha_{em}^{11} = \frac{{}_yE_1 + {}_{-y}E_1}{A}j,\ \alpha_{ee}^{23} = \frac{{}_xE_2 + {}_{-x}E_2}{A},\ \alpha_{ee}^{33} = \frac{{}_xE_3 + {}_{-x}E_3}{A},\\ &\alpha_{mm}^{11} = \frac{{}_yE_3 - {}_{-y}E_3}{A},\ \alpha_{me}^{23} = \frac{{}_xE_3 - {}_{-x}E_3}{A}j,\ \alpha_{me}^{33} = \frac{{}_{-x}E_2 - {}_xE_2}{A}j \end{aligned} \qquad (6)$$

4) $E_x\neq 0$, $H_z\neq 0$:

$$\alpha_{em}^{33} = \frac{{}_xE_3 + {}_{-x}E_3}{A}j,\ \alpha_{em}^{23} = \frac{{}_xE_2 + {}_{-x}E_2}{A}j,\ \alpha_{mm}^{23} = \frac{{}_{-x}E_3 - {}_xE_3}{A},\ \alpha_{mm}^{33} = \frac{{}_xE_2 - {}_{-x}E_2}{A} \qquad (7)$$

where $A = \dfrac{2\pi \cdot \varepsilon_0\mu_0 E_0 \cdot f^2}{R}$, E_0 is the incident electric field, f is the working frequency, ${}_xE_1$ is electric field radiated by the helix (left index denotes direction of wave investigation, right index denotes projection of electric field at this direction). All components of the tensors α_{ee}, α_{em}, α_{me}, α_{mm} of the one-turn and double-turn helices were calculated in the far field region at the frequency range 2-4 GHz using equations (4)-(7). For example, the components of magnetic polarizability tensor are shown for comparison in Fig. 1. The axial components of all polarizabilities of every helix differ less than to 10%. This confirms theoretical results obtained before. As one can see from Fig. 1, the component

α_{mm}^{33} is the biggest. The theoretical calculations for helices with the optimal shape have the same results.



a) b)

Fig.1 – a) Magnetic polarizability of the one-turn helix; b) Magnetic polarizability of the double-turn helix

The one-turn helix is excited in more wide frequency range than the double-turn helix is. This means the one-turn helix is more preferable for broadband cloaking by the wave flow method. The component α_{mm}^{33} of the double-turn helix is significantly bigger of other components in the tensor. Other polarizabilities tensors have the same property. Therefore, in some approximation one can take into account only one component, and not consider the whole tensor. This simplifies the calculations of properties artificial media very significantly. But this is not satisfied for the one-turn helix. The optimal shape helices are excited by arbitrary polarized incident electromagnetic wave. This simplifies process of placing the helices.

3. Conclusion

The obtained results make it possible to determine more exactly all components of polarizabilities tensors for the helices with the optimal shapes. Based on these results one can conclude that the double-turn helices are more preferable for some reasons and they can be used for cloaking by wave flow method.

5. Acknowledgement

This work was supported by the Belarusian Republican Foundation for Fundamental Research (project F11MS-005)

References

[1] I.V. Semchenko, A.L. Samofalov, S.A. Khakhomov, Radiation of Circularly Polarized Electromagnetic Waves by the Artificial Flat Lattice with Two-Turns Helical Elements, *Bianasotropics' 2004,* 10th International Conference on Complex Media and Metamaterials, Het Pand, Chent, Belgium, 22-24 September p. 236-239, 2004.

[2] A. N. Serdyukov, I. V. Semchenko, S. A. Tretyakov, A. H. Sihvola, *Electromagnetics of Bi-Anisotropic Materials* (Gordon and Breach Science), p. 337, 2001.

[3] L.D. Landau, E.M. Lifshitz, *The classical theory of fields,* Vol.2, Moscow, Nauka, p. 512, 1988.

