Radial photonic crystals for microwave operation

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Abstract

We analyze the properties of electromagnetic metamaterials with anisotropic constitutive parameters. Particularly, we analyze the so-called Radial Photonic Crystals, which are radially periodic structures verifying the Bloch theorem. This type of crystals can be designed and implemented in acoustics as well as in electromagnetism by using anisotropic metamaterials. They were originally proposed in acoustics, but similar functionalities are here proven in the electromagnetic domain together, for the first time, with an analysis of their behavior operating in the microwave regime. Finally, we present a complete discussion concerning their electromagnetic properties together with a practical design of such a microwave structure.

1. Introduction

Anisotropic phenomena have been largely investigated both in acoustics and electromagnetism. Control of anisotropic propagation is a key condition to allow a number of applications, which can cover a broad spectrum of areas from directive antennas to cloaking devices. In this context, periodic, semi-periodic and/or multilayered microstructures, depending also on the desired spatial configuration (1D, 2D or 3D), have been studied to satisfy different types of targets. These studies are usually based on the analysis of the desired propagation characteristics at the macroscopic level, and of the elementary constituent cells at the microscopic level. In particular, the selection of arrangement and unitary cell element geometry is inherently linked to the targeted spatial configuration, wave polarization, operation frequency and dispersive characteristics, and isotropic or anisotropic behavior, among others. We analyze and compare the wave propagation characteristics of the so-called Radial Wave Crystals (RWCs), [1], or Radial Photonic Crystals (RPCs) in the electromagnetics field. This type of metamaterial-inspired microstructure has the original characteristic of being invariant under translation, by implementing radially dependent constitutive parameter functions in multilayered systems. Although these structures were first considered for acoustic waves, clear analogies can be established between both application fields, [2]. For a 2D configuration, the fundamental characteristic of these microstructures derives from the periodicity of the radial profile of the anisotropic constitutive parameters. In this work, this is first analyzed by means of the proper design equations, together with the dispersion diagram information and related propagation parameters. Then, finite size structures based on these concepts are studied, the excitation of RPC structures with line sources is assessed and some potential applications are described. Finally, a practical design of a sample microwave microstructure is performed taking into account some simplifications that allow the feasibility of the device. These limitations are imposed on the number of constitutive parameters and layers implemented. This paper ends with a conclusion section summarizing the main findings of this work.

2. Radial Wave Crystals for electromagnetic waves

Starting from the scalar Helmholtz wave propagation equation, for the radial part and anisotropic parameters and TM^z waves (E^z field), we have:

$$\left[-\frac{1}{r\varepsilon_z(r)}\frac{\partial}{\partial r}\left(\frac{r}{\mu_\theta(r)}\frac{\partial}{\partial r}\right)+\frac{q^2}{r^2\varepsilon_z(r)\mu_r(r)}\right]E_q(r)=\omega^2 E_q(r) \qquad (1)$$

where $\varepsilon_z(r)$, $\mu_\theta(r)$ and $\mu_r(r)$ are radially dependent constitutive parameter isotropic functions and q is a constant. Equation (1) is a Bessel equation of order q that admits solutions invariant under translations of type $r \rightarrow r + nd$, [3]. A further analysis of the problem allows calculating a dispersion relation for a multilayered structure made of alternating layers of types a and b, with periodicity $d = d_a + d_b$ as:

$$\cos Kd = \cos k_{aq}d_a \cos k_{bq}d_b - \frac{1}{2}\left[\frac{\hat{\mu}_{a\theta}k_{bq}}{\hat{\mu}_{b\theta}k_{aq}}+\frac{\hat{\mu}_{b\theta}k_{aq}}{\hat{\mu}_{a\theta}k_{bq}}\right]\sin k_{aq}d_a \sin k_{bq}d_b \qquad (2)$$

Figure 1(a) displays the radial dispersion diagram for a RPC with constitutive parameters verifying the previous conditions. Figure 1(b) shows a set of constitutive parameters allowing the implementation of a 'reduced' parameter RPC structure with 4 layers (only $\varepsilon_z(r)$ and $\mu_\theta(r)$ are implemented and $\mu_r(r) = 1$). This reduced profile set behaves and is equivalent to the one of the ideal structure of Fig. 1(a).

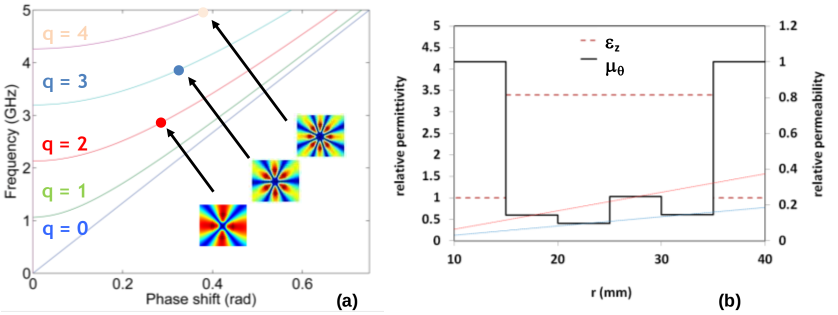

Fig. 1: (a) Dispersion diagram for the first 5 modes of a Radial Wave Crystal. The RPC is made from a multi-layered structure of alternate layers of types a and b with constitutive parameters (μ_{ra}^{-1}, $\mu_{\theta a}$, ε_{za}^{-1}) = (0.347r/d, 0.08r/d, 0.143r/d) and (μ_{rb}^{-1}, $\mu_{\theta b}$, ε_{zb}^{-1}) = (0.5r/d, 0.04r/d, 0.1r/d) and periodicity parameter $d = d_a + d_b$. Field patterns correspond to the design in the right panel; (b) Reduced model parameters for a four layers practical implementation as a function of the radial distance (center of the RPC located at r = 0 mm). Angular permeability (solid line) $\mu_\theta(r)$ follows a stair-like profile bounded by layers a and b analytical parameter functions (dotted lines); permittivity (dashed line) ε_z = 3.4 for all the RPC shell; radial permeability μ_r (not plotted) is constant and equal to that of the background (μ_r = 1).

3. Practical realization of a Radial Photonic Crystal

For a practical implementation of the constitutive parameters we have selected a unitary cell composed of a resonant split ring resonator (SRR) particle. This unitary component has reduced bianisotropy effects compared for example with other SRRs. This is important due to the fact that it is not desired that the array of resonators implementing the $\mu_\theta(r)$ function interferes with the $\mu_r(r)$ function (which in our case should be neutral $\mu_r(r)$ = 1). Generally, the permeability of an array of SRRs follows a Lorentz-like model with the resonant frequency separating positive and negative values of effective permeability. With the proper design of the geometric dimensions of the SRRs it is possible to define the permeability response at a specific frequency (the design frequency). An optimization process is required to match the desired permeability and permittivity values at the desired operation frequency (3.8 GHz, q =3 resonance in our case). Figure 2 displays a fabricated device based on parameters of Figure 1(b), together with the unit cell and the relevant geometric dimensions. We intend to verify the interaction of a line source in close location to the device.

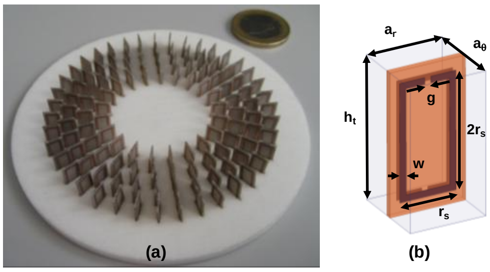

Fig. 2: (a) Fabricated 4 layers prototype made of Split Ring Resonators and a Rohacell ($\varepsilon_r \approx 1$) foam support. (b) Unit cell configuration to implement the angular permeability profile with characteristic dimensions.

The real part of the E-field patterns in the region around the RPC are displayed in Fig. 3. The RPC is centered on the origin of the radial coordinate system and the line source is located at position $x_0 = y_0$ = 95 mm. Figure 3(a) shows the result for an analytical model simulation (Comsol software). Figure 3(b) shows the simulation results for the SRR implemented RPC (cut plane plot of 3D simulation in Ansoft HFSS software), while Figure 3(c) shows the measured response of the fabricated device. Color scale represents real part of E-field with same range in all plots. From this comparison, it is clear that the q = 3 mode is excited in the RPC shell at the predicted resonance frequency, f = 3.80 GHz. Field maxima and minima are closely comparable in both simulations and the measurement. Orientation of the mode pattern towards the source is observed. Within the RPC shell, overall the field patterns closely follow the 'envelope' corresponding to the q = 3 field pattern.

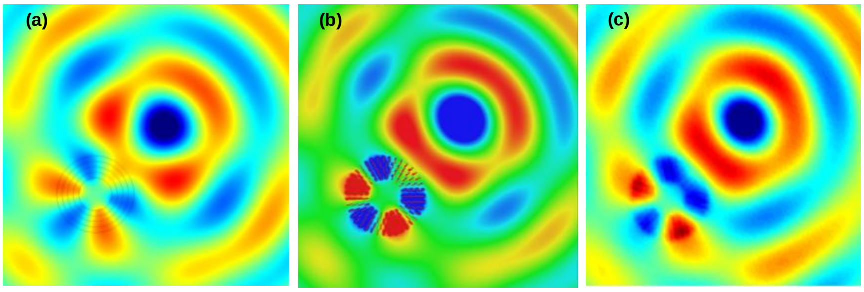

Fig. 3: Real part of the E-field of a line source externally illuminating a RPC shell at 3.8 GHz (a) analytical definition of the RPC constitutive parameters (Comsol simulation); (b) ab initio simulation of the RPC as in Fig. 2(a) (HFSS simulation); (c) measured results on the fabricated device.

4. Conclusion

Radial photonic crystals are 'very ordered' microstructures that have been analyzed by means of their electromagnetic constitutive parameters profiles showing their possible application as resonant structures. Their combination with radiation sources allows the analysis of the interactions and points out possible applications for example as beam-forming shells. Their application as frequency and position sensors can also be anticipated. The behavior of these microstructures was demonstrated with a prototype device operating at 3.8 GHz, whose response showed excellent agreement with our model.

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