Casimir-Lifshitz interactions in a one-way waveguide

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Abstract

We investigate the Casimir-Lifshitz interaction in a strongly nonreciprocal system: a waveguide filled with a hypothetical metamaterial representing a moving medium. We implement a one-way mode of propagation in such a waveguide and study how this may affect the Casimir force exerted on a body placed in it.

1. Introduction

Since the discovery of the Casimir effect [1] in 1948, this remarkable physical phenomenon has been under attention of physicists dealing with quantized fields. Being experimentally observable on a macroscopic scale, the electromagnetic Casimir effect is one of the most profound manifestations of the quantum nature of light, or, more generally, of the electromagnetic field. Unlike phenomena associated with emission or absorption of propagating light quanta — photons — the Casimir effect is rooted in such a state of quantum field that is characterized with no real particles — the ground state. The energy of the electromagnetic field does not vanish even in this state, which is a consequence of the uncertainty principle. Moreover, under the usual quantization framework, the energy of the electromagnetic quantum fluctuations in a cavity — the zero point energy — diverges to infinity! Various renormalization schemes exist to deal with this difficulty.

Nevertheless, the known observable phenomena in Casimir physics are due to a finite part of the zero point energy that explicitly depends on a set of macroscopic state parameters that may be changed externally. For example, in the case of the fluctuating electromagnetic field in a gap between two material bodies such important state parameters are the distance between the bodies, the angles defining the mutual orientation of the bodies, etc. Besides changes in geometry, it is evident that the electromagnetic states are very much affected by the background material in which the interacting bodies reside. The same applies to the electromagnetic properties of the bodies themselves. For instance, metal-dielectric bodies of simple shapes typically attract in vacuum, while there can be situations in which the same bodies repel when placed in a dielectric fluid. When the Casimir interaction happens *within a metamaterial* even more dramatic changes may occur, as it was demonstrated in our previous works [2, 3].

2. Theory and Discussion

In this work we ask what happens if the material that mediates the interaction by zero-point fluctuations is *strongly nonreciprocal*. Such a situation is very interesting from a theoretical point of view, because in a nonreciprocal medium the forward and backward directions of light propagation are not equivalent. Moreover, with magnetized metamaterials it is even possible to realize the so-called *one way waveguide* in which the photons travel only along a single direction. May we expect the Casimir forces between two objects inside such a waveguide to violate the third Newton law? May we extract energy and linear momentum out of the energetically-unlimited reservoir of zero-point fluctuations in such a structure?

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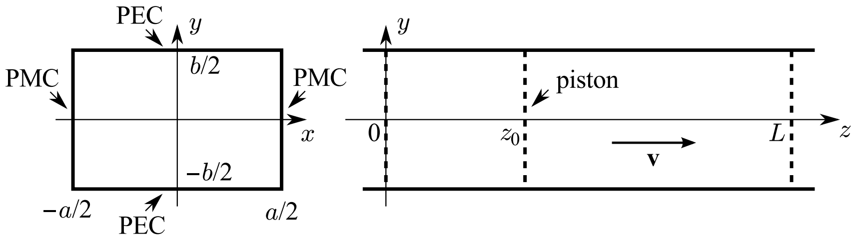

Fig. 1: Geometry of a waveguide filled with a moving dielectric fluid. Left: the cross-section of the waveguide. The waveguide is formed by two PEC plates at $y = \pm b/2$ and two PMC plates at $x = \pm a/2$ (the cross-section shape and the boundary conditions at the side walls are not of a particular importance for the effect that we study). Right: the section of the waveguide in the yz-plane. The waveguide is closed with two stationary walls at $z = 0$ and $z = L$. A wall that can slide along Oz (a piston) is placed at $z = z_0$. The walls are assumed penetrable by the fluid, but impenetrable by the electromagnetic field.

In order to answer these intriguing questions we consider a geometry shown in Fig. 1. This is a waveguide filled with a non-dispersive fluid that moves with the velocity v along the the waveguide axis (the z-axis). Alternatively, one may consider the same waveguide filled with a metamaterial that implements the constitutive relations of a moving medium. We close the waveguide at two selected cross-sections with fixed walls that strongly reflect the electromagnetic waves, but practically do not obstruct the fluid flow (one may imagine such walls as meshes of hair-thin wire, much like grids in vacuum tubes). We also place a sliding wall (a piston) with the same properties in between the other two stationary walls.

We are interested in the pressure exerted on the piston by the electromagnetic quantum fluctuations in the cavity formed by the waveguide walls and the piston (we consider only the electromagnetic part of the force and neglect any possible friction effects). For that we first solve for the eigenwaves supported by the waveguide (for details, see [4]) and show that at a fixed frequency the oppositely propagating eigenwaves in such waveguide may have largely different phase (and also group) velocities.

This is demonstrated in Fig. 2(left) in which we plot the phase velocities of the waves that co-propagate and counter-propagate with respect to the flow. At $v = 0$ both waves have equal velocities, however, when the flow velocity increases the counter-propagating wave slows down, while the co-propagating one speeds up. This is nothing more than just the well-known drag of radiation by the moving matter. However, something interesting happens at the point where the fluid velocity matches the velocity of light in the same medium at rest: $v = c/n$. At this point — which is the velocity threshold of Cherenkov emission — the counter-directed wave stops propagating, because its phase and group velocities approach zero. Thus, one may realize that this case is precisely the case of the one-way waveguide that we are interested in.

Let us note, however, that for the velocities *slightly below* the Cherenkov limit, the propagation of waves in the direction opposite to the medium flow is not really forbidden, and, moreover, it can be shown that the density of counter-propagating modes (per unit of frequency and volume) diverges to infinity as the velocity approaches c/n. However, in practice these modes cannot be excited because they are associated with extremely low phase velocities and extremely large wave vectors, and hence it is difficult to transfer energy to these modes and ensure the conservation of momentum. In other words, these modes are highly mismatched from any realistic source, and thus in practice only modes co-propagating with the medium can be excited near the Cherenkov threshold.

The Casimir energy of the fluctuating field inside a cavity can be found by adding up ground-state energies of all the electromagnetic modes in the cavity. Physically, in our system there are two such cavities (because the piston splits the structure in half) which can be considered separately. In [4] we consider the standing waves formed in these cavities and obtain an expression for the zero-point energy that explicitly depends on the velocity of the moving fluid. From this result it is immediately found that when the fluid velocity approaches the Cherenkov threshold — which is the required condition of one-

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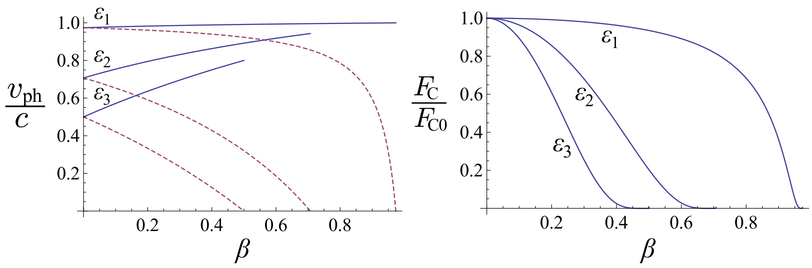

Fig. 2: (left) Normalized phase velocity $v_{\rm ph}/c$ of the TEM waves propagating in the waveguide filled with the moving fluid as a function of $\beta = v/c$ (the range of the plots is restricted to velocities below the Cherenkov's threshold). Solid curves: phase velocity of the wave co-propagating with the fluid flow. Dashed curves: phase velocity of the counter-propagating wave. The velocities are calculated for the following values of the dielectric constant of the fluid: $\varepsilon_1 = 1.0548$, $\varepsilon_2 = 2$, and $\varepsilon_3 = 4$. (right) Casimir force acting on the piston in a PEC waveguide with $a \times a$ cross-section filled with the moving fluid as a function of $\beta = v/c$. The force is calculated at $z_0 = a$ and normalized to the same force at $\beta = 0$. The three curves correspond to the three different values of the dielectric constant of the fluid.

way propagation in the considered waveguide — the Casimir energy in the waveguide *vanishes*, as well as the Casimir pressure on the piston does. For a piston in a square waveguide with PEC walls this is illustrated in Fig. 2(right), from which one may see that Casimir pressure on the piston rapidly decreases when v approaches c/n.

To explain this phenomenon one may argue that when the fluid moves at velocity equal to the speed of light in the same medium at rest, the electromagnetic field gets trapped within the moving matter so that it becomes impossible for the waves to propagate back and forward inside the waveguide and form a standing wave (i.e., a mode). It can be also shown that the characteristic frequency of any chosen mode tends to zero in this limit, which also explains why the Casimir energy vanishes at the Cherenkov threshold.

3. Conclusions

Thus, we may conclude that the Casimir pressure exerted on the piston simply vanishes in the limit of one-way propagation. Therefore, the net force produced by the quantum fluctuations of the electromagnetic field is zero in this case, and the conservation laws for energy and momentum are not violated, even though the wave propagation in the direction opposite to the fluid flow is forbidden.

The results of this work apply also qualitatively to the one-way waveguides implemented with metamaterials based on magnetized ferrites. However, because such materials are typically very dispersive, the one-way operation may occur in these systems only within a narrow band of frequencies. For the reasons outlined above, the virtual photons with frequencies within this band do not contribute to the zero-point energy. Based on this, one may expect a reduction in the value of the Casimir energy in these structures, and a respective reduction in the value of the Casimir force.

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