Pulling particles backward using a forward propagating beam

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Abstract

Photons carry a positive momentum of $\hbar k$ along its propagating direction. Thus one may expect light to "push" on objects standing on its path due the scattering force. Indeed, without an intensity gradient, it is counter intuitive for a light beam to pull particle backward. In this talk, we show the possibility for a forward propagating light beam to exert a backward scattering force that pulls a particle backward. We further show that it is the maximization of forward scattering via the interference of the radiation multipoles that is responsible for the backward forces. The necessary condition to achieve a pulling optical force requires both the particle and beam properties. This optical pulling adds an additional freedom to optical micromanipulation.

1. Introduction

Can the scattering force of a forward propagating beam pull a particle backward? A photon carries a momentum of $\hbar k$, so one may expect light to push against any object standing in its path. However, light can indeed “attract” in some cases. For example, if light is focused to a spot, small particles will be attracted towards it due to the gradient force. But it is probably more appropriate to say that the gradient force “grabs” rather than “pulls”, as the particle will remain stable in the trap after being drawn to the focus. Here, we discuss another possibility—a backward scattering force which is always opposite to the propagation direction of the beam so that the beam keeps on pulling an object towards the light source without reaching an equilibrium point [1,2].

In the absence of intensity gradient, using a light beam to pull a particle backwards is counter intuitive. The underlining physics is the maximization of forward scattering via interference of the radiation multipoles. We further show that by optimizing the geometry of the particle, one can significantly enhance the optical pulling force.

2. Optical forces from a propagation invariant beam

For simplicity, we shall consider a class of beam calls propagation invariant beam, which has zero intensity gradient along the propagation axis. Such beam exerts no gradient force on the particle, so all the forces exerted on the particle are scattering force. This considerably simplified our discussion. However, we stress that optical pulling force can also occurs for beams other than the propagation invariant beam. We shall consider the Bessel beam:

$$\overline{E} = E_0 e^{ik_z z + im\phi} \begin{Bmatrix} \left[im\alpha J_m\left(k_\perp \rho\right)/\rho + ik_z \beta k_\perp J'_m\left(k_\perp \rho\right)/k \right]\hat{\rho} \\ -\left[k_z m\beta J_m\left(k_\perp \rho\right)/k\rho + \alpha k_\perp J'_m\left(k_\perp \rho\right) \right]\hat{\phi} \\ +\left[\beta k_\perp^2 J_m\left(k_\perp \rho\right)/k \right]\hat{z} \end{Bmatrix}, \tag{1}$$

where (ρ,ϕ,z) are the cylindrical coordinates, $k_\perp = k\sin\theta_0$, $\alpha = \eta_{\mathrm{TE}} ik/k_\perp^2$, $\beta = \eta_{\mathrm{TM}} k e^{i\eta}/k_\perp^2$, η is the relative phase between transverse magnetic (TM) and transverse electric (TE) waves, J_m is the Bessel function of order *m*, and J'_m is the derivative of the Bessel function with respect to its argument. A propagation invariant beam consists of plane wave components whose *k*-vector makes an angle θ_0 with the propagation axis. The projection of incident photons' momentum on the propagation axis is $\hbar k\cos\theta_0$. In a scattering event, the photon is scattered into another direction, and on average, the momentum of the scattered photon is $\hbar k\langle\cos\theta\rangle$. It can be shown that the optical force along the beam axis exerted on a spherical particle by a propagation invariant beam is given by

$$F = W_{sca} c^{-1}\left[\cos\theta_0 - \langle\cos\theta\rangle\right], \tag{2}$$

where W_{sca} is the rate of photon energy scattering, and *c* is the speed of light. It can be inferred from (2) that for a plane wave, $\cos\theta_0 = 1$, so optical pulling force is impossible. However, for propagation invariant beam like (1), $\cos\theta_0 < 1$, i.e. the *z*-component photon momentum decreases with increasing θ_0. Suppose there exists a particle that can scatter all the incident light into the forward direction (i.e. $\theta = 0^{\mathrm{o}}$). Then (2) will become

$$F = W_{sca} c^{-1}\left[\cos\theta_0 - 1\right] < 0, \tag{3}$$

which is less than zero, i.e. the force is pointing in a direction opposite to beam propagation direction. This argument heuristically shows the possibility of having an optical pulling force, which can exist if the scattered wave is emitted mainly in the forward direction. Nevertheless, it is by no means obvious that (2) can be less than zero for a real particle. Therefore we have to go into rigorous calculation.

3. Rigorous calculation demonstrating the optical pulling force

To analytically demonstrate optical pulling force, we derived, from the Lorentz force, the multipole expansion of the time-averaged optical force up to electric quadrupole order:

$$\vec{F} = \vec{F}_{\text{incident}} + \vec{F}_{\text{interference}}, \tag{4}$$

where

$$\begin{aligned} \vec{F}_{\text{incident}} &= \vec{F}_{\vec{p}} + \vec{F}_{\vec{m}} + \vec{F}_{\ddot{Q}_e} + \dots, \\ \vec{F}_{\text{interference}} &= \vec{F}_{\vec{p}\vec{m}} + \vec{F}_{\ddot{Q}_e\vec{p}} + \dots, \end{aligned} \tag{5}$$

and

$$\vec{F}_{\vec{p}} = \tfrac{1}{2}\mathrm{Re}\{\nabla\vec{E}^{*}\bullet\vec{p}\}, \qquad \vec{F}_{\vec{m}} = \tfrac{1}{2}\mathrm{Re}\{\nabla\vec{B}^{*}\bullet\vec{m}\},$$
$$\vec{F}_{\vec{Q}_e} = \tfrac{1}{4}\mathrm{Re}\{\nabla\nabla\vec{E}^{*}:\overleftrightarrow{Q}_e\}, \qquad . \qquad (6)$$
$$\vec{F}_{\vec{p}\vec{m}} = -\tfrac{k^4}{12\pi\varepsilon_0 c}\mathrm{Re}\{\vec{p}\times\vec{m}^{*}\}, \quad \vec{F}_{\vec{Q}_e\vec{p}} = -\tfrac{k^5}{40\pi\varepsilon_0}\mathrm{Im}\{\overleftrightarrow{Q}_e\bullet\vec{p}^{*}\},$$

It can be shown that $\vec{F}_{\text{incident}}$ is the amount of momentum removed by the particle from the beam, which is positive definite. Therefore, any negative force must be coming from $\vec{F}_{\text{interference}}$. Since $\vec{F}_{\text{interference}}$ is the recoil force, as long as there are more photons being scattered into the forward direction than the backward direction, it can negative. We have performed explicit analytical and numerical calculation that demonstrates the ability of $\vec{F}_{\text{interference}}$ to be negative and dominate over $\vec{F}_{\text{incident}}$ to achieve optical pulling force. We note that every terms in $\vec{F}_{\text{interference}}$ consists of a product of two multipoles. Physically, $\vec{F}_{\text{interference}}$ is induced by the fact that the radiation multipoles interfere with each other to produce anisotropic scattering, and this anisotropic scattering induces a recoiled force.

In our previous work [1], we demonstrated that one can exert an optical pulling force on a spherical particle. However, the optical pulling force actually requires highly anisotropic scattering. It then seems that spherical particle is not the most appropriate particle in terms of achieving optical pulling force. As a matter of fact, for spherical particles, a relatively large θ_0 ($>50^{\circ}$ in typical case) is needed. However, if one is allowed to tailor the particle morphology, the situation is completely different. For example, as shown in Fig. 1, our two dimensional calculation for a rectangular block shows that one can observe optical pulling force for θ_0 as small as 13°.

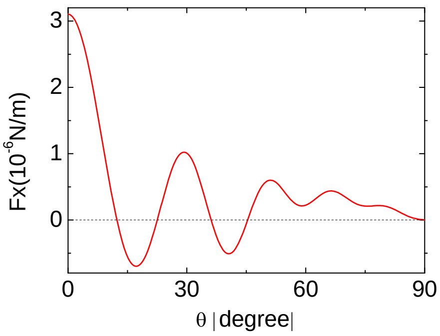

Fig. 1 Optical force acting on a rectangular block (with length $4\mu m$, width $0.82\mu m$, and the third dimension being infinite) by a pair of plane wave whose *k*-vectors make an angle θ with the beam propagation axis.

4. Conclusion

From rigorous numerical simulation and analytical calculation, it is established that light can indeed pull a particle backward using scattering force, and we have also determined the condition in which such optical pulling force can be observed. The optical pulling force is enabled by the interference enhanced forward scattering of radiation multipoles induced by the incident beam.

References

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