Casimir Forces in Chiral Metamaterials

C. M. Soukoulis[1,3], R. Zhao[2,3], E. N. Economou[1] and Th. Koschny[3]

[1]Institute of Electronic Structure and Laser, Foundation for Research and Technology, Hellas (FORTH), P.O. Box 1385, Heraklion, Crete, Greece
[2]Blackett Lab, Dept. of Physics, Imperial College, London, SW7 2AZ United Kingdom
[3]Ames Lab and Dept of Physics and Astronomy, Iowa State University, Ames, IA, USA
soukouli@iesl.forth.gr ; soukoulis@ameslab.gov

Abstract

We use the extended Lifshitz theory to study the behavior of the Casimir forces between chiral metamaterials and regular metamaterials. We have shown that the chirality, if strong enough, is of critical importance to reduce the Cassimir attractive force and can find new designs to obtain repulsive Casimir forces. We will review chiral metamaterials and find some analytical expression for the strengths of electrical permittivity, magnetic permeability and chirality for passive materials.

1. Introduction

The so-called Casimir effect [1-3], in its original version, is the theoretical prediction that two closely-spaced plane-parallel mirrors will be mutually attracted, due to the modification of the electromagnetic mode structure between the mirrors. This attractive force comes about from the zero-point energy associated with the modes; the total energy decreases as the plates are brought together. Since this force is due to zero-point energy, the force persists even at absolute zero temperature. Casimir forces observed experimentally [3] in nature have almost always been attractive, and have rendered nanoscale and microscale machines inoperable by causing their moving parts to permanently stick together. *This has been a long-standing problem that scientists working on such devices have struggled to overcome.* In this context, the original theory was extended to include the EM response of the two plates (as characterized by the permittivities and the permeabilities), as well as the presence of a fluid material between the two plates. Moreover, the expression for the force (or the interaction energy) was rewritten in terms of the reflection amplitudes [4] for EM waves propagating from the medium in between towards each of the plates. Still, if the medium in between is the vacuum or air, the force continues to be attractive. *Since the frequency dependence of the permittivity $\varepsilon(\omega)$ and the permeability $\mu(\omega)$ of the metamaterials can be tailored to some extent, it is only natural to employ them in the hope their reflection amplitudes will be such that repulsive Casimir forces may appear under certain conditions.* This possibility, in combination with the improvements in measuring small forces [3] near surfaces and progress in the ability to complete complex numerical computations [5] efficiently, has led to an intensified effort in studying the Casimir force. Furthermore, as nanotechnology progresses, the need for nanolevitation becomes more pressing, leading to increased efforts and demands for manipulating the Casimir forces. These demands have been met partly by computations and some measurements [1-3].

Very recently in a series of papers [6-8], our group has employed homogeneous chiral MMs to reduce Casimir attraction or even to reverse it, if the chirality coefficient is sufficiently strong. Most of the above

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ideas have been criticized [9,10] for using the effective medium theory beyond the range of its applicability or unrealistic values of the chirality parameter. More specifically, it was stated in homogeneous chiral MMs, its passivity requirement sets restrictions on the size of the chirality strength for single resonance frequency-dependence [9(a); 9(b) provide a reply to this comment]. These restrictions do not allow the particular model used in the first of our papers [6] to reach the critical chirality value for repulsion. However, no explicit result has been obtained for the maximum possible chirality in the general case of homogeneous chiral materials possessing several resonances. More serious is the point raised by the manuscript [10], according to which the effective medium theory employed in our first paper [6] fails for distances, *d*, between the Casimir plates of the order 10 nm or so (where the Casimir force is strong), if the size of the unit cell of the MM is larger than 10nm. *The whole community plan to spend substantial time to determine realistic-based MMs (chiral and plasmonics), analyzed without using the effective medium theory and the homogeneity of the MMs that can achieve the goal to control Casimir forces*.

2. Chirality in Metamaterials

To design a chiral metamaterial, we usually start from a non-chiral resonator that is much smaller than the relevant wavelength. This resonator may provide a resonant electric or magnetic response. Then we modify the spatial structure of the resonator such that it responds to the driving field with both an electric and a magnetic dipole moment, simultaneously. An illustrative example is given in Fig. 1. Here, the non-chiral building block is a simple SRR. The SRR has a magnetic resonance determined by the loop inductance of the metal ring and the capacitance of the gap. A resonant current oscillates around the ring and charges accumulate across the gap. Although the SRR can couple to an external electric field across it's gap, in the usual configuration the SRR couples to an external (time dependent) magnetic field perpendicular to the ring plane. The magnetic field induces a resonant ring current, such that, in turn, the SRR responds to the

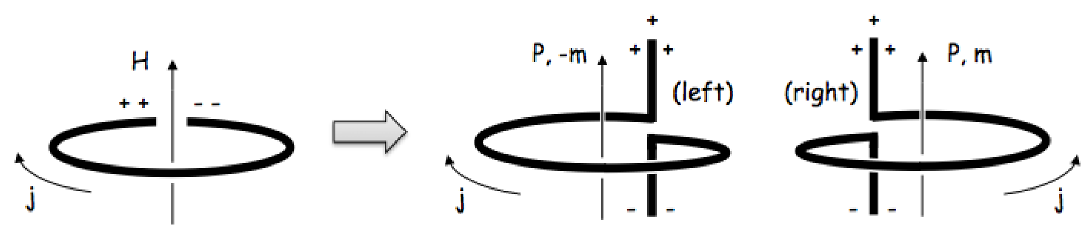

Fig. 1: Chirality by shape. Deforming a non-chiral single-ring SRR out of plane makes it chiral.

external magnetic field with a resonant magnetic dipole moment. This is the magnetic response of the SRR. The charge accumulation across the gap also causes an electric dipole moment across the gap, which leads to an additional bi-anisotropic response of the SRR. In non-chiral metamaterial designs this is usually explicitly compensated by the design (e.g. opposed ring double-layer SRRs), small enough to be neglected (narrow gap SRR or overleaf-capacitor SRR), or ignored as irrelevant as it cannot interfere with the propagating mode because of the mode polarization and orientation with respect to the exciting field (e.g. in 1D and 2D metamaterials for fixed polarization). Now, we make use of this additional electric dipole moment to construct a chiral metamaterials. We bend the two ends of the SRR, where the charges accumulate, upward on one side, downward on the other, out of the plane of the SRR ring as illustrated in Fig. 1. *In this geometry the electric dipole moment of the SRR now aligns with the direction of the SRR's magnetic dipole moment.* Dependent on which of the ends will be bent upwards, the electric and magnetic dipole moments will be parallel or anti-parallel to each other up to a phase shift of 90 degrees (the electric moment is proportional to the accumulated charge; the magnetic moment is proportional to the current, which is the time derivative of the charge, hence gets an additional factor $\pm i\omega$).

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The resonant structure we so constructed is chiral because it cross-couples the electric and magnetic fields in the same direction. An incident magnetic field perpendicular to the ring plane couples to the SRR loop, drives the ring current of the SRR's LC resonance, and caused both, a magnetic and an electric dipole moment response of the SRR in the same direction as the incident magnetic field. Simultaneously, an incident electric field perpendicular to the ring plane couples by polarizing the out-of-plane ends of the SRR, which in turn drives the ring current of the SRR's LC resonance, and leads to both, electric dipole and magnetic dipole response in the direction of the incident electric field. This is exactly the dependence of each of the electric and magnetic polarizations on both, the electric and the magnetic field as described by the chiral constitutive relations given above. The chiral resonator design we just obtained is known as "Chiral Omega-particle". It owes it's chirality to the chiral, "spiral like" shape.

The response functions of this idealized Omega-particle can be given analytically [see ref. 7 and 11]:

$$\varepsilon(\omega)=1+\frac{\Omega_\varepsilon\omega_0^2}{\omega_0^2-\omega^2-i\omega\gamma};\ \mu(\omega)=1+\frac{\Omega_\mu\omega^2}{\omega_0^2-\omega^2-i\omega\gamma};\ \kappa(\omega)=\frac{\Omega_\kappa\omega_0\omega}{\omega_0^2-\omega^2-i\omega\gamma}$$

The pre-factors $\Omega_\varepsilon, \Omega_\mu\ and\ \Omega_\kappa,\ with\ \Omega_\kappa^2=\Omega_\varepsilon\Omega_\mu$ [7,11], are geometry dependent constants. This condition limit the possibility of obtaining large chirality [7,11]. It is obvious that the resonance in the chiral material leads to a diagonal electric and a diagonal magnetic response in $\varepsilon(\omega)$ and, $\mu(\omega)$ respectively, in addition to the resonant chiral response in $\kappa(\omega)$. This is a general property of chiral media: as the chirality cross-couples electric and magnetic fields, any chiral response to an electric field will create an magnetic polarization, hence an electric field; the chiral response of which will create an magnetic moment again such that the material also has a diagonal magnetic response. The diagonal electric response follows analogously. Depending on the particular design, the diagonal electric response or the diagonal magnetic response may be more pronounced and either may become negative (i.e. $\varepsilon(\omega)<0$or $\mu(\omega)<0$) if the resonance is strong enough. Both will always be present. It is therefore impossible to have "purely electric" or "purely magnetic" chirality.

3. Reduction of the Casimir attractive force and possible repulsion Casimir force

In ref. 6, we have used the effective medium theory for chiral metamaterials and we have obtained repulsive Casimir force given in Fig. 2 with a strong chirality. As long as the first point raised in the comment [9a), namely that $\Delta=\frac{1}{c^2}\left[\mathrm{Im}(\varepsilon)\mathrm{Im}(\mu)-(\mathrm{Im}(\kappa))^2\right]$ **(1)** has to be positive for passive materials, we would like to point out the following: In our PRL [6] we have discussed the condition, $\Omega_\varepsilon\Omega_\mu=\Omega_\kappa^2$ **(2)** which is related to the condition for Δ given above. In our recent publication [7] on the strength of the chirality, we have proven that $\Omega_\varepsilon\Omega_\mu=\Omega_\kappa^2$, which seems to limit the possibility of obtaining large chirality Ω_κ. In the parameters that we have used in our PRL [6], Δ remains positive if Ω_κ is less than $0.032\,\omega_R$, while our relation (2) $\Omega_\kappa^2=\Omega_\varepsilon\Omega_\mu=(\omega_R)(0.001\omega_R)=0.001\omega_R^2$, which gives the same condition $\Omega_\kappa=0.032\omega_R$. In order to satisfy the conditions (1) and (2) one needs to increase the $\Omega_\varepsilon\ and\ \Omega_\mu$, and therefore the chirality will increase and therefore Ω_κ will be large and will reduce the attractive Casimir force. In Figure 3, we plot the reduction of attractive Casimir force when the chirality increases and satisfies the conditions (1) and (2), and, therefore, it does not violate the passivity of the material.
In the future we plan to spend substantial time to determine realistic-based MMs (chiral and plasmonics), analyzed without using the effective medium theory and the homogeneity of the MMs that can achieve the goal to control Casimir forces. By developing novel designs and analysing them; more specifically, we do

not only compare the chiralities of the experimentally realizable new and old designs, but also check whether or not these presented designs can lead to a repulsive Casimir force in the framework of the effective approximation.

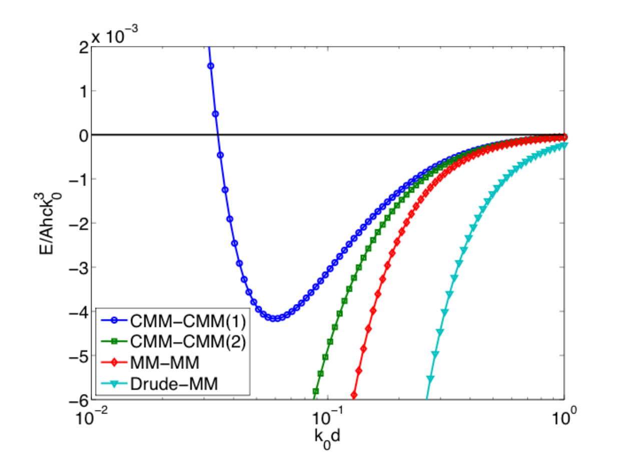

Fig. 2:. Casimir interaction energy per unit area E/A versus k_0d. The repulsive Casimir force is the blue line with strong chirality. This figure reproduced with permission from ref. 6.

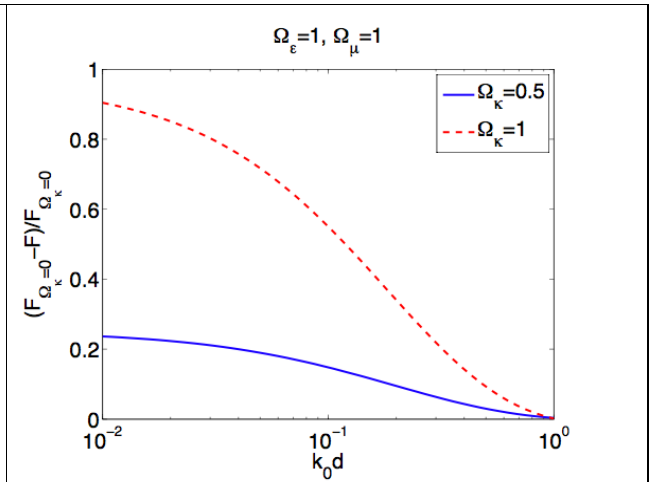

Fig. 3. Reduction of the attractive Casimir force for two different strengths of chirality. The reduction is 20% for $\Omega_\kappa = 0.5$ and 90% for $\Omega_\kappa = 1$. This figure reproduced with permission from ref. 9b.

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