Reflection/transmission of obliquely incident TE waves from metasurfaces homogenized via surface susceptibility models

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Abstract

In this paper, analytical S-parameters expressions for periodic metasurfaces of uniaxially mono-anisotropic scatterers under TE wave incidence, are extracted. Unlike existing formulas, the derived expressions include the effect of off-diagonal terms in the susceptibility matrix, which represent a weak form of spatial dispersion at the lattice level. The results for various angles of incidence are compared to numerical simulations, in order to reveal the importance of the proposed generalization.

1. Introduction

Metafilms or metasurfaces, the 2-D counterparts of 3-D metamaterials (MTMs), are usually implemented by periodically arranging electrically small scatterers on a surface [1]. Since metasurfaces are easier to fabricate compared to bulk MTMs, as they require less physical space, they frequently lead in more affordable and less lossy components. These advantages make them suitable for a wide range of applications like absorbers, polarization rotators, and subwavelength resonators, to name only a few [2, 3].

Unlike 3-D MTMs, metasurfaces cannot be characterized in terms of effective constitutive parameters, due to the ambiguity in the definition of their effective thickness. Instead, they can be properly described by means of surface susceptibilities, which associate their electromagnetic field response to the surface polarization currents induced in their plane [4]. In order to be considered as characteristic parameters, surface susceptibilities of a metasurface should be independent on the incidence angle [5]. Moreover, they can be utilized for the prediction of the far-field response of a metasurface for arbitrary incidence angle, even after a change of lattice characteristics, without needing any further full-wave simulations.

Recently, an analytical methodology for the extraction of surface susceptibilities for metasurfaces of uniaxially mono-anisotropic scatterers under oblique TE wave incidence has been developed [6]. For their calculation, only the polarizabilities of the scatterers and the dynamic, angle-dependent intraplanar interaction coefficients [7] are required. In this paper, the generalized expressions for the reflection and transmission coefficients of periodic metasurfaces illuminated by obliquely incident TE plane waves are systematically derived. The resulting formulas incorporate all the existing susceptibility terms, including the off-diagonal elements that have been lately reported [8]. The relevance of these magneto-electric coupling terms, which are non-vanishing even for mono-anisotropic inclusions, due to the interparticle retardation effects at the grid level, will be studied in the numerical results section.

2. Generalized reflection and transmission coefficients

Assume a metasurface located at the $z = 0$ plane, consisting of uniaxially mono-anisotropic scatterers arranged according to a rectangular lattice with periods a and b along the x- and y-axis, respectively, as illustrated in Fig. 1(a). For a TE$_z$-polarized plane wave that impinges on the metasurface, the normalized expressions for the incident, reflected, and transmitted field in vectorial form can be written as

$$\mathbf{f}^{\rm inc} = [\varepsilon_0 E_x^{\rm inc}, c^{-1} H_y^{\rm inc}, c^{-1} H_z^{\rm inc}]^T = \varepsilon_0 E_0 e^{-j(k_y y + k_z z)} [1, \cos\theta, -\sin\theta]^T, \tag{1a}$$

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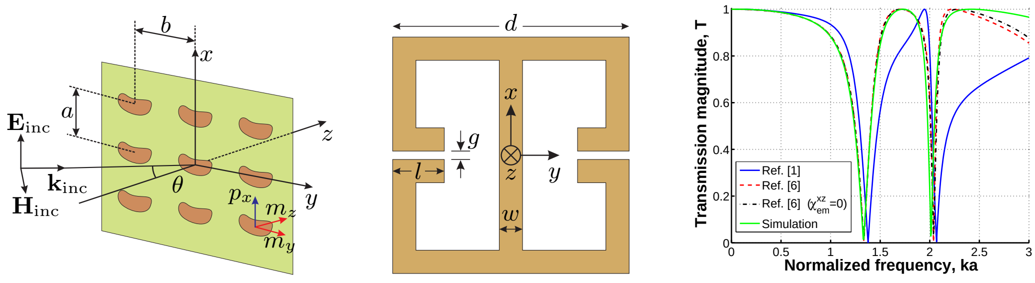

(a) (b) (c)

Fig. 1: (a) A metasurface with arbitrarily-shaped scatterers at the $z = 0$ plane, (b) geometry of an ERR with $d = 5$ mm, $l = 1.2$ mm, $w = 0.5$ mm, and $g = 0.3$ mm, and (c) transmission coefficient for $\theta = 45°$.

$$\mathbf{f}^{\rm ref} = [\varepsilon_0 E_x^{\rm ref}, c^{-1}H_y^{\rm ref}, c^{-1}H_z^{\rm ref}]^T = \varepsilon_0 R E_0 e^{-j(k_y y - k_z z)}[1, -\cos\theta, -\sin\theta]^T, \tag{1b}$$

$$\mathbf{f}^{\rm tran} = [\varepsilon_0 E_x^{\rm tran}, c^{-1}H_y^{\rm tran}, c^{-1}H_z^{\rm tran}]^T = \varepsilon_0 T E_0 e^{-j(k_y y + k_z z)}[1, \cos\theta, -\sin\theta]^T, \tag{1c}$$

where R and T are the reflection and transmission coefficients to be determined, θ the angle between the wavevector $\mathbf{k}^{\rm inc}$ of the incident wave and the z-axis, and k_y, k_z the components of the wavevector along the respective axes. Following [6] and assuming electrically-small periodicities $(a, b < \lambda/2)$, the metasurface in the far field region is seen as a uniform surface with effective surface polarizations

$$\overline{\boldsymbol{\mu}}_s = [\overline{P}_{sx}, c^{-1}\overline{M}_{sy}, c^{-1}\overline{M}_{sz}]^T. \tag{2}$$

Surface polarizations are related to the average total field at the metasurface plane, $\overline{\mathbf{f}}$, through

$$\overline{\boldsymbol{\mu}}_s = [\chi]\overline{\mathbf{f}} = \frac{1}{2}(\mathbf{f}^{\rm inc} + \mathbf{f}^{\rm ref} + \mathbf{f}^{\rm tran}), \tag{3}$$

where $[\chi]$ is the surface susceptibility matrix of the metasurface. Finally, the desired expressions for R and T are acquired by substituting (1)-(3) into the generalized sheet transition conditions (GSTCs) that relate the electromagnetic fields on both sides of the metasurface [1], that read

$$\hat{\mathbf{z}} \times (\mathbf{H}|_{z=0^+} - \mathbf{H}|_{z=0^-}) = j\omega\overline{P}_{sx}\hat{\mathbf{x}} - \hat{\mathbf{z}} \times \nabla\overline{M}_{sz}, \tag{4a}$$

$$\hat{\mathbf{z}} \times (\mathbf{E}|_{z=0^+} - \mathbf{E}|_{z=0^-}) = -j\omega\mu_0\overline{M}_{sy}\hat{\mathbf{y}}. \tag{4b}$$

Inversion of the resulting system of equations yields (5a) and (5b), provided at the bottom of the page. These expressions are generalizations of previously reported ones, since they include the non-zero, off-diagonal χ_{em}^{xz} element of the susceptibility matrix. This term, although frequently negligible, can play a significant role in the proper description of specific metasurfaces, as shown in the following section. It is obvious that, if $\chi_{em}^{xz} = 0$, these expressions transform to the corresponding expressions of [1, 5].

3. Numerical results

In this section, (5a) and (5b) are applied in various metasurfaces for the estimation of their scattering parameters. All the simulations were performed via the Finite Element Method (FEM), considering an infinite array of particles located on the xy-plane, while the lattice periods were set to $a = b = 7$ mm.

$$R = \frac{-\frac{jk_0}{2\cos\theta}\left(\chi_{ee}^{xx} - \chi_{mm}^{yy}\cos^2\theta + \chi_{mm}^{zz}\sin^2\theta - 2\chi_{em}^{xz}\sin\theta\right)}{1 - \frac{k_0^2\chi_{mm}^{yy}}{4}\left(\chi_{ee}^{xx} + \chi_{mm}^{zz}\sin^2\theta - 2\chi_{em}^{xz}\sin\theta\right) + \frac{jk_0}{2\cos\theta}\left(\chi_{ee}^{xx} + \chi_{mm}^{yy}\cos^2\theta + \chi_{mm}^{zz}\sin^2\theta - 2\chi_{em}^{xz}\sin\theta\right)} \tag{5a}$$

$$T = \frac{1 + \frac{k_0^2\chi_{mm}^{yy}}{4}\left(\chi_{ee}^{xx} + \chi_{mm}^{zz}\sin^2\theta - 2\chi_{em}^{xz}\sin\theta\right)}{1 - \frac{k_0^2\chi_{mm}^{yy}}{4}\left(\chi_{ee}^{xx} + \chi_{mm}^{zz}\sin^2\theta - 2\chi_{em}^{xz}\sin\theta\right) + \frac{jk_0}{2\cos\theta}\left(\chi_{ee}^{xx} + \chi_{mm}^{yy}\cos^2\theta + \chi_{mm}^{zz}\sin^2\theta - 2\chi_{em}^{xz}\sin\theta\right)} \tag{5b}$$

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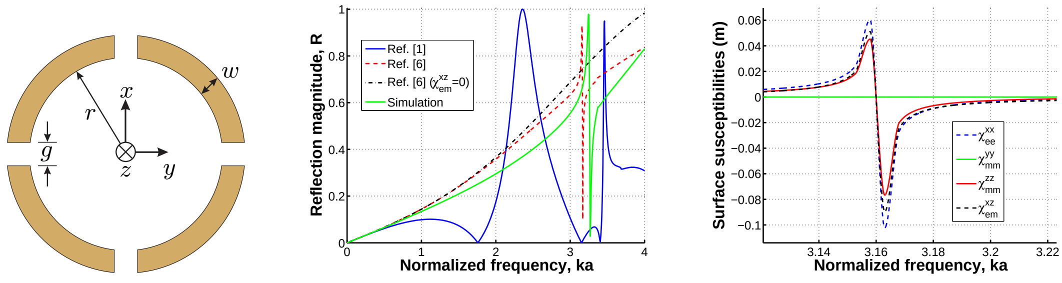

(a) (b) (c)

Fig. 2: (a) Geometry of a FSR with $r = 3$ mm, $w = 0.4$ mm, and $g = 0.2$ mm, (b) reflection coefficient for $\theta = 60°$, and (c) comparison of the surface susceptibility terms of the metasurface.

The results from two different surface susceptibility models, those of [1] and [6], have been inserted in (5a) and (5b) and compared with full-wave simulations for various oblique incidences. For the electric ring resonator (ERR) of Fig. 1(b), the transmission coefficient for a $45°$ incident TE wave [Fig. 1(c)] is obtained from the aforementioned techniques, which appear to be in very good agreement with the numerical simulations. The χ_{em}^{xz} term is practically negligible in that case, since ignoring it does not affect the accuracy of the method in [6]. On the other hand, for the four-split ring (FSR) of Fig. 2(a), the reflection coefficient for a $60°$ incident wave [Fig. 2(b)] reveals the importance of the bi-anisotropic term χ_{em}^{xz}, which significantly affects the resonance at $ka = 3.26$. This is highlighted also in Fig. 2(c), where the comparison of the surface susceptibilities, obtained from [6], reveals its comparable amplitude with respect to the diagonal susceptibility terms, indicating the potential significance of its insertion into (5).

4. Conclusion

In this paper, generalized formulas for the transmission and reflection coefficients of periodic metasurfaces under oblique incidence, have been introduced. Simulation results show that the magneto-electric coupling term χ_{em}^{xz}, previously ignored in homogenization models of metasurfaces, can potentially prove significant in the proper prediction of the scattering parameters, like in the case of a four-split resonator.

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