Local framework for nonlocal wire media

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Abstract

In this presentation we outline a framework applicable to wire media formed by sets of nonconnected wires. In this framework, the macroscopic material response is described with more independent degrees of freedom than usual. This allows formulating a system of local field equations with which both boundary value problems and problems of radiation of the sources embedded into wire medium can be solved efficiently.

1. Introduction

Wire media (WM) are metamaterials formed by arrays of long conductors — metallic wires — typically embedded into a dielectric host. In uniaxial WM all the wires are oriented along the same axis, while in WM with more than a single set of wires there can be more than one available direction for the conductors. The ability to conduct electric current along sets of wires is the most characteristic feature of WM that has a great impact on the macroscopic electromagnetic response. Namely, the strong spatial dispersion in WM in the limit of very long wavelengths can be shown to result from redistribution of electric charge along the wires, due to which the wires become locally charged. This effect is purely quasi-static [1] and is analogous to the drift-diffusion phenomena in semiconductors which also lead to the emergence of spatially dispersive (nonlocal) effects.

On the other hand, the wires in WM are similar to the conductors in transmission lines, so that each set of wires along a given direction can be understood as a multiwire transmission line (TL). In such an analogy the nonlocality of WM is due to the fact that TL modes propagate along the wires without diffraction and efficiently "connect" separate regions within the metamaterial, so that the medium response at a given point depends on the field values at distant points. Under a quasi-static approximation, the relation between the full electromagnetic description of WM and the TL analogy can be readily established. This has been done in our previous works (see [1, 2]).

2. Theory and Discussion

Here, we consider WM composed of three mutually orthogonal disconnected sets of wires, oriented along x, y, and z axes, respectively. In this case the quasi-static treatment [1] results in the following system of macroscopic field equations (the Maxwell equations) coupled to a number of TL-type equations for the internal degrees of freedom associated with the TL modes propagating along the independent sets of wires:

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu_h \mathbf{H}(\mathbf{r}), \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\varepsilon_h \mathbf{E}(\mathbf{r}) + \mathbf{J}(\mathbf{r}) + \mathbf{J}^{\rm ext}(\mathbf{r}), \tag{2}$$

$$\partial_i I_i(\mathbf{r}) = -j\omega C_i \varphi_i(\mathbf{r}), \tag{3}$$

$$\partial_i \varphi_i(\mathbf{r}) = -(j\omega L_i + Z_i^w) I_i(\mathbf{r}) + E_i(\mathbf{r}) + V_i^{\rm ext}(\mathbf{r}), \tag{4}$$

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where ε_h and μ_h are the parameters of the host; the index $i = x, y, z$ labels the sets of wires and the respective components of the radius vector $\mathbf{r}$, with the notation $\partial_i \equiv \partial/\partial r_i$; $I_i(\mathbf{r})$ and $\varphi_i(\mathbf{r})$ represent the internal degrees of freedom associated with the current and the additional potential [1] within a given set of wires, $\mathbf{J}(\mathbf{r}) = \sum_i (I_i(\mathbf{r})/A_i)\hat{\mathbf{e}}_i$ is the total average current density, with A_i being the unit cell area associated with each set of wires, L_i, C_i, and Z_i^w are the effective inductance, capacitance and self-impedance [1] (for example, ohmic resistance) per unit length of a wire in a given set. Note that there is no implicit summation over repeating indices in (3) and (4). An important feature of the system (1)–(4) is that it allows introducing a new type of source which has the physical meaning of the external electromotive force per unit length of the wires $V_i^{\rm ext}$ [3].

Let us stress that Eqs. (1)–(4) constitute a *local* framework for WM in which the effects of spatial dispersion are described by introducing a few internal degrees of freedom into the model. In our model these functions, $I_i(\mathbf{r})$ and $\varphi_i(\mathbf{r})$, have a clear physical meaning: one being related to the wire current and the other to the additional potential at the wire surface acquired due to redistribution of the electric charge along the wire [1]. The locality of this framework is seen from the fact that the material response represented by the expressions on the right-hand side of Eqs. (1)–(4) (without the source terms) does not depend on the spatial gradient ∇ or any higher derivatives of the electromagnetic fields, $I_i(\mathbf{r})$, and $\varphi_i(\mathbf{r})$. Therefore, spatially inhomogeneous or layered WM can be treated within this framework rather trivially: it is enough to assume that the material parameters ε_h, μ_h, C_i, L_i, and Z_i^w are functions of the position vector $\mathbf{r}$. Note that in our framework the boundary conditions (BCs) at abrupt interfaces follow naturally from Eqs. (1)–(4) under the assumption of finiteness of the fields, currents and potentials at a boundary. In the traditional nonlocal models that do not deal with the internal degrees of freedom directly the same BCs appear in an obfuscated form of the so-called additional BCs (ABCs) [2].

When dealing with uniform and unbounded WM it is, of course, possible to eliminate the internal degrees of freedom from the system (1)–(4), which leads in the end to the usual description of WM based on the spatially dispersive permittivity tensor $\overline{\overline{\varepsilon}}(\omega, \mathbf{k})$. Indeed, in the spatial Fourier domain $\partial_i \leftrightarrow -jk_i$, and from (3) and (4) it follows that for the plane waves $J_i = -j\omega\varepsilon_h(E_i + V_i^{\rm ext})k_{p,i}^2/(k_h^2 - j\xi_i k_h - k_i^2/n_i^2)$, where $k_h = \omega\sqrt{\varepsilon_h\mu_h}$, $k_{p,i}^2 = \mu_h/(L_iA_i)$, $n_i = L_iC_i/(\varepsilon_h\mu_h)$, and $\xi_i = (Z_i^w/L_i)\sqrt{\varepsilon_h\mu_h}$. When this expression for the current density is substituted into the Maxwell equations (1) and (2), it results in a diagonal effective permittivity $\overline{\overline{\varepsilon}}(\omega, \mathbf{k})$ with the components $\varepsilon_{ii}(\omega, k_i) = \varepsilon_h[1 - k_{p,i}^2/(k_h^2 - j\xi_i k_h - k_i^2/n_i^2)]$. Additionally, there appear (nonlocal) source terms proportional to $V_i^{\rm ext}$.

However, the formalism based on the nonlocal dielectric function can be applied without any assumptions only to unbounded uniform WM. As soon as there is an interface at which the wires are cut, or the properties of the host change abruptly, one has to assume certain ABCs at the interface, because without such additional conditions the boundary value problems become underdetermined. It is commonly accepted that such ABCs cannot be derived from the Maxwell equations without knowing the microstructure of the metamaterial — the fact which for sure puzzles many researchers working in the field. However, from the point of view of our local framework such a situation is a mere consequence of eliminating physically significant internal degrees of freedom! Therefore, it is not surprising that the information contained within the spatially dispersive permittivity $\overline{\overline{\varepsilon}}(\omega, \mathbf{k})$ and the macroscopic Maxwell equations is, in general, not complete to solve boundary value problems, or problems involving nonuniform WM.

The local framework outlined above allows also for obtaining a closed-form expression for the Poynting vector in WM [3]. We do it for the case of a nondispersive host (ε_h and μ_h are real constants) and metallic wires described by $Z_i^w = j\omega L_i^{\rm kin} + R_i$, where $L_i^{\rm kin}$ is the effective kinetic inductance and R_i is the effective ohmic resistance per unit length of wires. These parameters are related to the plasma frequency ω_p and the collision frequency Γ of the Drude model of wire metal and they also may be assumed nondispersive. Therefore, we may rewrite Eqs. (1)–(4) in the time domain replacing $j\omega \to \partial/\partial t$, and derive the following conservation law: $\nabla \cdot \mathbf{S} = -\frac{\partial w}{\partial t} - P_{\rm loss} + P_{\rm ext}$, where

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} + \sum_i \frac{\varphi_i I_i}{A_i}\hat{\mathbf{e}}_i, \quad w = \frac{\varepsilon_h \mathbf{E}^2}{2} + \frac{\mu_h \mathbf{H}^2}{2} + \sum_i \frac{1}{A_i}\left(\frac{C_i\varphi_i^2}{2} + \frac{L_i^{\rm tot} I_i^2}{2}\right), \tag{5}$$

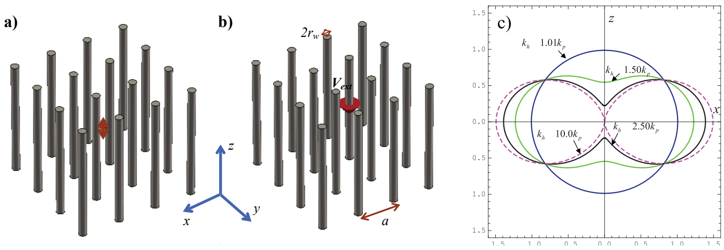

Fig. 1: (From Ref. [3]) Uniaxial WM formed by a square lattice of metallic wires oriented along the z-direction. (a) Excitation by a short vertical dipole. (b) Excitation by a voltage source. (c) Polar plot of the directive gain of a short vertical dipole in the uniaxial WM at different frequencies of operation.

$$P_{\text{loss}} = \sum_i \frac{R_i I_i^2}{A_i}, \qquad P_{\text{ext}} = \sum_i \frac{V_i^{\text{ext}} I_i}{A_i} - \mathbf{E} \cdot \mathbf{J}^{\text{ext}}, \tag{6}$$

where $L_i^{\text{tot}} = L_i + L_i^{\text{kin}}$. The vectorial quantity $\mathbf{S}$ is understood as the Poynting vector in WM, and P_{ext} as the volume density of the power transferred by the external sources to the medium. In the absence of loss, i.e. when $R_i = 0$, the term w is univocally identified with the density of stored energy. In contrast, if loss is present, then it is generally impossible to separate the energy storage rate from the energy loss rate when a metamaterial is considered macroscopically. However, if the microstructure of a metamaterial is known, the stored energy can be found from a consistent physical model that fully describes the processes within a unit volume of the metamaterial. Thus, if we assume that the Drude model is such a consistent model for the dynamics of the free electron plasma in metals, then w in (5) preserves the meaning of the stored energy density even when $R_i > 0$. In this case, the quantity P_{loss} has the physical meaning of an instantaneous power loss density.

3. Examples and Conclusions

As an example of application of our local framework to real problems, we consider radiation of elementary sources embedded into a uniaxial WM [3] (Fig. 1). We assume that there is a pair of such sources: an electric dipole oriented along the wires (along the z-axis) and a voltage source inserted into the wires. In the local framework, such radiation problem is formulated by Eqs. (1)–(4) with delta-functional source terms $\mathbf{J}^{\text{ext}}$ and V_z^{ext} concentrated at the origin. Because the problem has cylindrical symmetry and all the currents are along the z-axis, the problem may be solved by introducing the Hertz potential for the electromagnetic fields, and an auxiliary potential describing the wave propagation governed by Eqs. (3) and (4). In this case the system (1)–(4) reduces to a pair of second-order differential equations for the potentials, which can be solved with standard techniques, and the fields can be found from the potentials. For the field created by the vertical dipole an interesting result is obtained: the radiation pattern of such a dipole within WM with perfectly conducting (PEC) wires approaches the pattern of a hypothetical isotropic radiator when the frequency is close to the WM plasma frequency [Fig 1(c)]. We also find that the dipole source and the voltage source produce very different radiation patterns. The radiation from the voltage source in a PEC WM is characterized with a nondiffractive pattern of an infinite directivity, with all the radiated power concentrated in a vicinity of a vertical line passing through the source.

References

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