Effective parameter extraction of SRR structures by means of first-principles homogenization techniques

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Abstract

In this paper, the effective parameters of typical SRR structures are extracted via first-principles homogenization techniques. Each metamaterial unit-cell is assumed to be an electrically small scatterer, and its polarizabilities are obtained through dynamic approaches. Then, the retrieved polarizabilities are used in the homogenization formulas to derive the desired effective medium parameters.

1. Introduction

During the past years, homogenization has emerged as an important aspect of metamaterial analysis. Due to its contribution in both the characterization and the design of metamaterial structures, it became necessary to develop accurate and reliable techniques for the extraction of effective material parameters. Amid them, the Nicolson-Ross-Weir (NRW) method has an increasing popularity owing to its simplicity and relatively acceptable results. However, in certain cases (e.g. resonance bands), it produces several non-physical artifacts that violate the passivity and causality conditions. Therefore, the research in the area of homogenization has focused on microscopic field analysis in order to mitigate the prior defects.

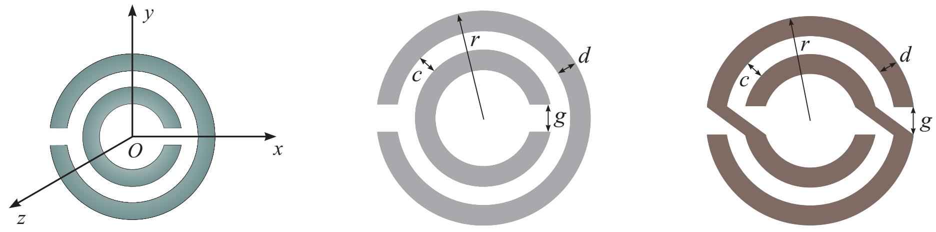
In this paper, a recent first-principles homogenization approach [1] is implemented for the extraction of the effective parameters of several well-known split ring resonators (SRRs) [2]. Essentially, the specific method has been, so far, applied only in magnetodielectric spheres, whose polarizabilities can be acquired via analytical formulas. Herein, and unlike existing procedures, we utilize the polarizabitilies retrieved by dynamic approaches [3, 4] to successfully derive the effective parameters of the SRRs. Various results are provided in order to certify the validity of the method for different useful setups.

2. Polarizability retrieval and homogenization approach

Stemming from the Clausius-Mossotti formulas [5], the first-principles homogenization approach utilizes the polarizabilities of a single resonant particle to retrieve the bulk effective parameters of 3D periodic arrangements of the same scatterer. The concept is based on the replacement of each scatterer by appropriate sets of electric and magnetic dipole moments and the analytical calculation of their interaction, via the proper 3D coefficients [4]. In this context, for a single edge-coupled SRR (EC-SRR), with reference to the orientation of Fig. 1(a), the polarizability matrix is written as

$$[\boldsymbol{\alpha}] = \begin{bmatrix} \alpha_{ee}^{xx} & 0 & 0 \\ 0 & \alpha_{ee}^{yy} & \alpha_{em}^{yz} \\ 0 & -\alpha_{em}^{yz} & \alpha_{mm}^{zz} \end{bmatrix}, \tag{1}$$

where α_{ee}^{xx}, α_{ee}^{yy}, α_{em}^{yz}, and α_{mm}^{zz} are the electric-electric, electric-magnetic, and magnetic-magnetic polarizabilities of the scatterer, correspondingly. The elements of (1) can be obtained from approximate expressions [2], or from a dynamic approach based on the S-parameters of a 2D array with the same scatterers, as in [3]. Note that for non-bianisotropic scatterers, the above extraction procedure reduces to the well-known method of [4]. The polarizabilities, so obtained, can be incorporated in the homogenization formulas provided in [1]. Hence, the effective parameters of the SRR are obtained as



(a) (b) (c)

Fig. 1: (a) Orientation of the SRRs, (b) EC-SRR, and (c) NB-SRR ($c = d = g = 0.25$ mm and $r = 3$ mm).

$$\varepsilon^{yy}_{r,\text{eff}} = 1 + [a^{-3}(\alpha'_m - C_{int})]/\Lambda, \qquad \mu^{zz}_{r,\text{eff}} = 1 + [a^{-3}(\alpha'_e - C_{int})]/\Lambda, \tag{2}$$

$$\chi^e_{\text{eff}} = c_0^{-1} a^{-3} \alpha'_{em}/\Lambda, \qquad \chi^o_{\text{eff}} = c_0^{-1} a^{-3} C'_{em}/\Lambda, \tag{3}$$

with $\Lambda = (\alpha'_e - C_{int})(\alpha'_m - C_{int}) - C'^2_{em} + \alpha'^2_{em}$, $\alpha'_e = \alpha^{zz}_{mm}[\alpha^{yy}_{ee}\alpha^{zz}_{mm} + (\alpha^{yz}_{em})^2]^{-1}$, $\alpha'_m = \alpha^{yy}_{ee}[\alpha^{yy}_{ee}\alpha^{zz}_{mm} + (\alpha^{yz}_{em})^2]^{-1}$, $\alpha'_{em} = \alpha^{yz}_{em}[\alpha^{yy}_{ee}\alpha^{zz}_{mm} + (\alpha^{yz}_{em})^2]^{-1}$, $C_{int} = C_{3\text{D}} - a^{-3}k_0^2/(\beta^2 - k_0^2)$, and $C'_{em} = C_{em,3\text{D}} - a^{-3}\beta k_0/(\beta^2 - k_0^2)$, with $C_{3\text{D}}$ and $C_{em,3\text{D}}$ the 3D lattice co-field and cross-field interaction coefficients, respectively. The parameters of (2) and (3) satisfy the constitutive relations for the average fields

$$\mathbf{D}_{av} = \varepsilon_{\text{eff}}\mathbf{E}_{av} - (\chi^e_{\text{eff}} + \chi^o_{\text{eff}})\hat{\boldsymbol{\beta}} \times \mathbf{H}_{av}, \qquad \mathbf{B}_{av} = \mu_{\text{eff}}\mathbf{H}_{av} - (\chi^e_{\text{eff}} - \chi^o_{\text{eff}})\hat{\boldsymbol{\beta}} \times \mathbf{E}_{av}, \tag{4}$$

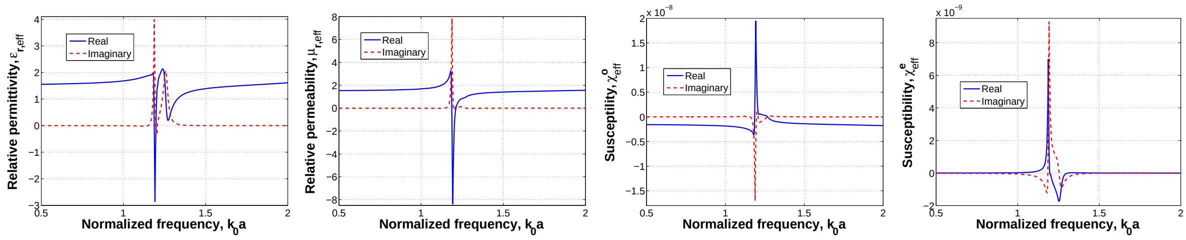
Wavenumber β, required for completing the homogenization procedure, is found by employing the NRW method [6], since it can adequately predict the refractive index of a medium. Thus, β can is derived from $\beta^2 = \omega^2\mu_{eq}\epsilon_{eq} \Rightarrow \beta^2 = n^2_{\text{NRW}}k_0^2$.

3. Parameter retrieval and discussion

To verify the above methodology, three common SRR designs are investigated, namely the free-standing arrays of EC-SRRs and NB-SRRs [Figs 1(b)-1(c)], and an array of NB-SRRs imprinted on a lossless FR-4 substrate. For all the designs, the polarizabities are extracted in terms of the dynamic approach described in [3] and are employed for the retrieval of the effective parameters from (2) and (3).

For the EC-SRR, the results of Fig. 2 reveal a mu-negative (MNG) behavior right above $k_0a = 1.19$. However, the homogenization procedure produces some unexpected artifacts prior to the resonance, such as the violation of passivity condition $\varepsilon''_{r,\text{eff}} < 0$. This issue can be possibly attributed to the error in the estimation of the polarizabilities or the refractive index values, as obtained from the NWR method. Also, one should be sceptical about the homogenization attempt in the band-gap zone, where the dimensions of the unit cell are not electrically small, due to the high values of the effective parameters [1].

Next, we consider the NB-SRR of Fig 1(c) whose results, plotted in Fig. 3, indicate a MNG behavior above the resonance occurring at $k_0a = 1.17$. Nevertheless, it can be also detected that $\epsilon'_{r,\text{eff}}$ receives negative values, as in the EC-SRR case. Since the odd part of the effective susceptibility, χ^o_{eff}, exhibits a resonance at the same frequency, the epsilon-negative (ENG) behavior displayed in Fig. 3(a) is suppressed, because of the opposite signs of $\varepsilon_{r,\text{eff}}$, $\mu_{r,\text{eff}}$ and χ^o_{eff} in (4). On the other hand, χ^e_{eff} is zero for this structure, due to its non-bianisotropic nature ($\alpha^{yz}_{em} = 0$).



(a) (b) (c) (d)

Fig. 2: Effective parameters for the EC-SRR of Fig. 1(b) in a cubic lattice of $a = 10$ mm.

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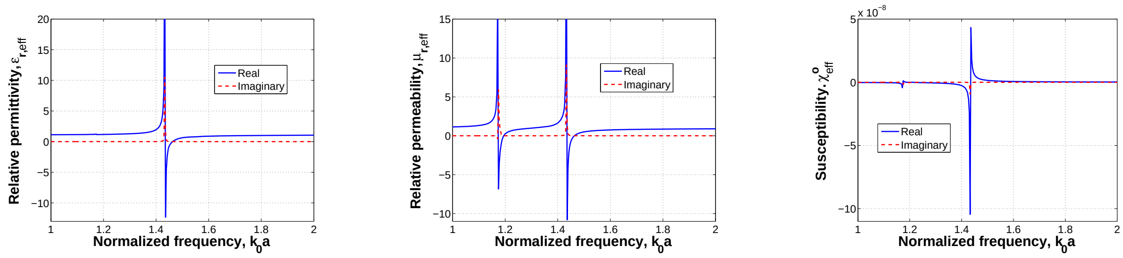
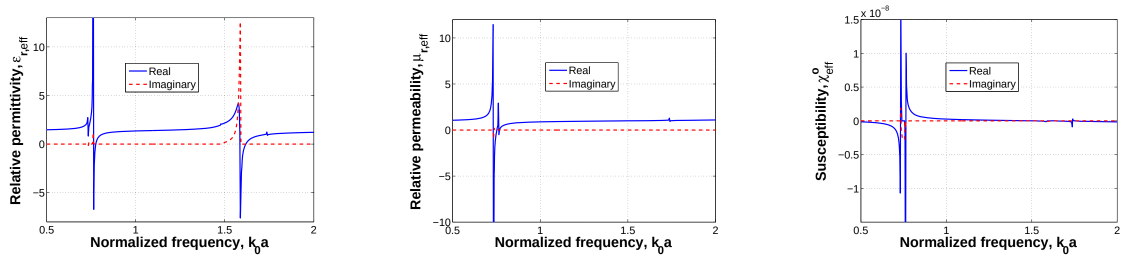


(a) (b) (c)

Fig. 3: Effective parameters for the NB-SRR of Fig. 1(c) in a cubic lattice of $a = 10$ mm.



(a) (b) (c)

Fig. 4: Effective parameters for the NB-SRR of Fig. 1(c), imprinted on a 1.5 mm-thick FR4 substrate.

Finally, a NB-SRR of the same dimensions, imprinted on a lossless 1.5 mm-thick FR-4 slab ($\varepsilon_r = 4.4$) is examined. For the polarizability extraction procedure, both the NB-SRR structure and the dielectric substrate are considered as a scatterer. Naturally, the addition of the dielectric substrate downshifted the resonant frequency to $k_0 a = 0.74$. Results are otherwise similar to those of the free-standing NB-SRR, where $\chi^e_{\text{eff}} = 0$ due to the negligible bianisotropy. Also, the χ^o_{eff} term suppresses the ENG behavior of Fig. 4(a) near the magnetic resonance, whereas the resonance at $k_0 a = 1.59$ is of electric nature.

4. Conclusion

In this paper, effective parameter retrieval for various SRR structures has been performed. The results are proven accurate and almost free of artifacts; thus, they can be safely used for metamaterial design processes. Future aspects of this work include further improvement of the developed algorithm, especially toward the more precise determination of the wavenumber β and the polarizability α^{yz}_{em}.

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