A scalable architecture for solid-state quantum metamaterials

R.D. Wilson[1], **M.J. Everitt**[1], **S. Savel'ev**[1] **and A.M. Zagoskin**[1]

[1]Department of Physics
Loughborough University
Loughborough, LE11 3TU, UK
email:R.D.Wilson@lboro.ac.uk

Abstract

Quantum metamaterials provide a promising potential test bed for probing the quantum-classical transition. We propose a feasible and scalable architecture for a solid-state quantum metamaterial. This consists of an ensemble of superconducting flux qubits inductively coupled to a superconducting transmission line. We make use of a quasi-classical model to study the transmission properties of the proposed architecture and we also discuss the possibilities for experimentation with this type of extended quantum system.

1. Introduction

Metamaterials are one of the fastest growing and most promising areas of research in optics and the extension of this paradigm into the quantum regime has opened up even more possibilities. Quantum metamaterials were first proposed by Rakhmanov *et al.* in [6] and they are composed of controllable unit elements, which maintain quantum coherence throughout the time it takes for an electromagnetic signal to propagate through the structure. Since then a number of textbook quantum optical phenomena have been demonstrated in this type of structure as proof of concept, *i.e.* resonance fluorescence [2], electromagnetically induced transparency [1], and amplification [3]. Other unusual effects such as quantum birefringence, where the quantum metamaterial is prepared in a superposition of states with different effective refractive indices, have also been postulated to be realisable with these devices [8]. This type of controllable quantum system is also interesting from a fundamental point of view as it allows us to directly probe the quantum-classical transition in a macroscopic system.

In this work, we propose a scalable and experimentally feasible architecture for a solid-state quantum metamaterial. The proposed metamaterial architecture consists of persistent-current superconducting flux qubits [5] which are inductively coupled to a superconducting transmission line, as shown in Fig. 1. We will then go on to discuss the transmission properties of the system, as well as the types of experiment that could be performed with it.

2. Theoretical model

We model the superconducting transmission line as a chain of inductively coupled LCR oscillators which are in turn inductively coupled to the flux qubits. The key advantage of this approach is that we can readily model the system in the semi-classical limit as well as the fully quantum mechanical regime. In the quasi-classical limit, the equation of motion for the coupled oscillators in the line is

$$C\ddot{\phi}\left(t\right)+\frac{1}{R}\dot{\phi}\left(t\right)+\frac{1}{L}\Lambda^{-1}\left(\phi\left(t\right)+M_{lq}\tilde{I}_{q}\left(t\right)\right)=\tilde{I}_{in}\left(t\right), \tag{1}$$

where $\phi\left(t\right)$ is the vector containing the fluxes of the oscillators in the line, $I_q\left(t\right)$ is a vector containing the qubit current expectation values and Λ is the dimensionless inductance matrix. Λ is a symmetric matrix

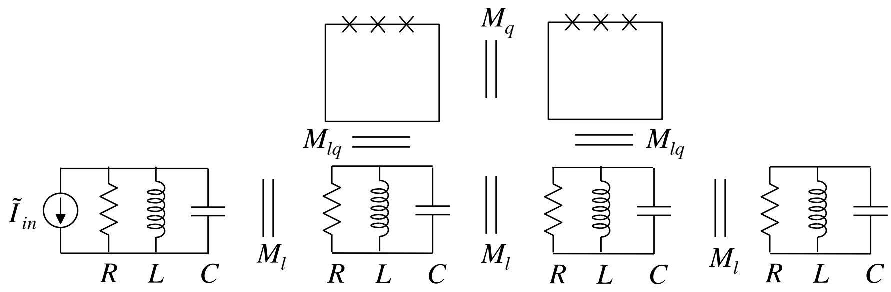


Fig. 1: Basic schematic of the active region of a 1D quantum metamaterial which consists of a superconducting transmission line with inductively coupled flux qubits.

that describes all the mutually inductive coupling between the oscillators. It has diagonal elements equal to unity, representing the self inductances of the oscillators, and non-zero off-diagonal elements equal to k_l when the two respective oscillators are coupled through a mutual inductance $M_l = k_l L$.

The equation of motion governing the state of the qubits in the system, $|\psi(t)\rangle$, will simply be the Schrödinger equation;

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}H(t)|\psi(t)\rangle. \tag{2}$$

Coupled flux qubits of this type can be described by a Hamiltonian of the form;

$$H(t) = -\sum_{i=1}^{N}\left(I_p\left(\phi_i + \tilde{\phi}_{xi} - \tfrac{1}{2}\Phi_0\right)\sigma_z^{(i)} + \Delta_i\sigma_x^{(i)}\right) + \sum_{ij} M_q I_p^2 \sigma_z^{(i)}\sigma_z^{(j)}, \tag{3}$$

where I_p is the persistent current flowing in qubits, $\tilde{\phi}_{xi}$ is the applied flux threading qubit i, M_q is the mutual inductance between the qubits and Δ_i is the qubit half energy splitting at the $\frac{1}{2}\Phi_0$ bias point, as in [7].

The equations of motion for the qubits and the transmission line are linked by the qubit current vector, I_q. The elements of the vector I_q whose respective oscillators are coupled to a qubit will be given by

$$I_{qi}(t) = \left\langle \hat{I}_i(t)\right\rangle = I_{pi}\langle\psi(t)|\,\sigma_z^{(i)}\,|\psi(t)\rangle, \tag{4}$$

otherwise the elements of I_q will be zero. The form of (1) and (3) allow us to construct a metamaterial system consisting of a network of any combination of coupled qubits and oscillators. However, initially we will restrict our analysis to a 1D chain of oscillators with qubits coupled at intervals along the chain.

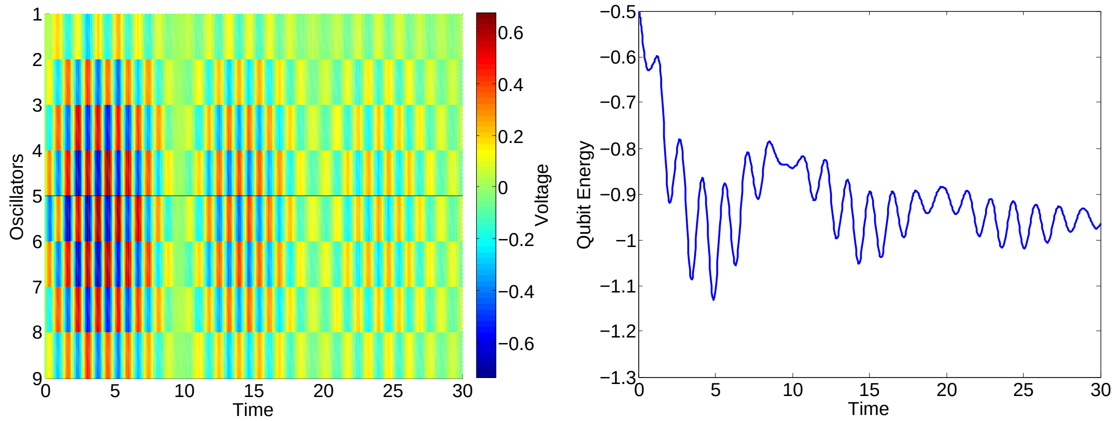
3. Initial results

We solve the equations of motion, (1) and (2), numerically in dimensionless form. Figure 2 shows an example of the results for a transmission line with a single qubit connected to the central oscillator node. We can see from Fig. 2(a) that the initial input pulse induces oscillations in the line that propagate back and forth along it and interfere with each other. Over time these oscillations are damped out by the resistance term in (1). From Fig. 2(b) it is evident that the qubit's energy expectation value oscillates over time. This implies that there is an exchange in energy taking place between the line and qubit as the waves propagate along the line; which is a clear indicator that the qubit is acting as a pointlike scatterer.

4. Conclusions and outlook

We have proposed a feasible and scalable architecture for a solid-state quantum metamaterial and a quasi-classical theoretical model that can be used to describe it. Initial testing has demonstrated that a qubit

ISBN 978-952-67611-2-1



(a) (b)

Fig. 2: (Colour online) Results for a transmission line of 9 identical oscillators with a single flux qubit coupled to the central oscillator and an inter-oscillator and line-qubit coupling constant of $k = 0.5$. The qubit begins in it's ground state and a current pulse is incident on oscillator 1 at $t = 0$. (a) Voltage along the transmission line as a function of time. The solid black line denotes the position of the qubit. (b) Energy expectation value of the qubit as a function of time. The results are presented in dimensionless units of the form $[t] = 1/\sqrt{LC}$, $[V] = \sqrt{LC}/\Phi_0$ and $[E] = \Phi_0^2/L$.

can act as a pointlike scatterer. This work will be extended to look at different configurations of this type of 1D quantum metamaterial and we will further explore how the interaction between the line and qubits affects the propagation of signals along the transmission line. In particular, we would like to determine the circuit parameters necessary to realise some of the unusual effects found in quantum metamaterials, such as quantum birefringence, as well as looking for new types of phenomena. Where possible, we will also compare the results of this quasi-classical model of a quantum metamaterial to those of a fully quantum mechanical model, where the transmission line is considered to be a chain of quantum harmonic oscillators.

References

[1] A. A. Abdumalikov, O. Astafiev, A. M. Zagoskin, Yu. A. Pashkin, Y. Nakamura, and J. S. Tsai. Electromagnetically Induced Transparency on a Single Artificial Atom. *Physical Review Letters*, 104(19), May 2010.

[2] O Astafiev, A M Zagoskin, A A Abdumalikov, Yu A Pashkin, T Yamamoto, K Inomata, Y Nakamura, and J S Tsai. Resonance fluorescence of a single artificial atom. *Science (New York, N.Y.)*, 327(5967):840–3, February 2010.

[3] O. V. Astafiev, A. A. Abdumalikov, A. M. Zagoskin, Yu. A. Pashkin, Y. Nakamura, and J. S. Tsai. Ultimate On-Chip Quantum Amplifier. *Physical Review Letters*, 104(18), May 2010.

[4] Devoret, M. H. Quantum Fluctuations in Electrical Circuits. *Fluctuations Quantiques / Quantum Fluctuations: Les Houches Session LXIII*, 1997.

[5] J E Mooij, T P Orlando, L Levitov, Lin Tian, Caspar H van der Wal, and Seth Lloyd. Josephson Persistent-Current Qubit. *Science*, 285(5430):1036–1039, 1999.

[6] A. Rakhmanov, A. Zagoskin, Sergey Savelev, and Franco Nori. Quantum metamaterials: Electromagnetic waves in a Josephson qubit line. *Physical Review B*, 77(14), April 2008.

[7] SHW Van der Ploeg, A. Izmalkov, M. Grajcar, U. Hubner, S Linzen, S Uchaikin, Th Wagner, A.Y. Smirnov, A.M. van den Brink, MHS Amin, and Others. Adiabatic quantum computation with flux qubits, first experimental results. *Applied Superconductivity, IEEE Transactions on*, 17(2):113–119, 2007.

[8] A.M. M Zagoskin. *Quantum Engineering: Theory and Design of Quantum Coherent Structures*. Cambridge Univ Pr, 2011.