# Plano-Planar lenses using $\varepsilon$ near-zero stacked waveguides at millimeter waves 

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#### Abstract

In this paper we aim to show how stacked cut-off waveguides emulating an $\varepsilon$ near-zero metamaterial can be used to generate efficient frequency-dependent plane-plane lenses. First, an array of cut-off waveguides is cut at $30^{\circ}$ to steer a plane wave that angle. Afterwards, a graded index lens designed to perform the same $30^{\circ}$ redirection with a flat metamaterial lens.


## 1. Introduction

Epsilon near-zero (ENZ) metamaterials have attracted the attention of the scientific community due to their salient features, such as energy squeezing and supercoupling [1-3]. For instance, it has been demonstrated that a channel with arbitrary cross-section filled with an ENZ metamaterial can be used to connect and match waveguides of different dimensions [1].

On the other hand, narrow hollow waveguides have been proposed as a realization of an ENZ media [1, 2]. In this work, we propose the design of an ENZ prism and an ENZ graded index lens based on arrangement of narrow channel waveguides following the ideas developed in [1-2]. Basically, a small phase progression is allowed inside each waveguide to provide the desired phase distribution at the output lens face. The study comprises two different designs: a prism which converts a normally incident plane wave into a plane wave at 30 degrees and a graded index lens with the same performance.

## 2. ENZ prism at $\mathbf{3 0}^{\boldsymbol{\circ}}$

To begin with, the unit cell is presented in Fig. 1(a) whose design parameters are: $t=5 \mathrm{~mm}, d_{y}=$ $2.5 \mathrm{~mm}, d_{x}=1 \mathrm{~mm}, h_{x}=0.1 \mathrm{~mm}$ and $h_{y}=1.5 \mathrm{~mm}$. The dimension $h_{y}$ is fixed to work at the cut-off wavelength of the fundamental mode $\mathrm{TE}_{01}: h_{y}=\lambda_{o} / 2$ where $\lambda_{o}$ is the wavelength at the operation frequency of 100 GHz ; whereas $h_{x}$ has been chosen to follow the rule of thumb $h_{x}=d_{x} / 10$ to achieve impedance matching [2].


Fig. 1: (a) Unit cell, perspective and front view. (b) Prism, perspective and top view.
Using this waveguide, the semi-prism is made by stacking waveguides up to a total number of 22, all of them with the same dimensions, except the length $t$ which varies from 5 mm to 17.7 mm in order to build a prism with an angle of 30 degrees in one of its faces. The final structure is shown in Fig.1(b).

The structure is simulated using CST Microwave Studio ${ }^{\text {TM }}$ with open boundary conditions on the lateral sides and magnetic walls at $\pm d_{y}$. The structure is excited using an $x$-polarized plane wave with normal propagation along $z$-axis. Figure 2 depicts the calculated magnetic field distribution $\left(H_{y}\right)$.


Fig. 2: Simulation results of the magnetic field $\mathrm{H}_{\mathrm{y}}$ : (a) Magnitude, (b) Phase and (c) $\mathrm{H}_{\mathrm{y}}$-Field.
The simulation results show how a plane wave at 30 degrees is shaped due to the cut made to the structure at the output face. Some unavoidable diffraction on the edges is observed, giving rise to some interference in the radiation pattern. Increasing the number of waveguides, this undesired effect would be minimized. However, it is clear that the operation of the prism is as expected: the phase fronts that emerge from an ENZ metamaterial follow the exit interface due to the phase velocity inside the medium is effectively infinite [3]. In the next section, we will achieve the same effect but with a graded index lens rather than a prism, using an array of waveguides with the same dimension $t$.

## 3. Graded Index Lens

The structure is shown in Fig.3. For this new design, 66 waveguides have been used and the reference waveguide is the first one from right to left. In the figure, $x_{i}$ is the distance between the first waveguide and each one of the other members of the array and $d_{i}$ is the excess distance of the wave emitted by each waveguide, taking as a reference the waveguide on the right and assuming that the wave at the output is at 30 degrees.


Fig. 3: Representation of the new design using the same length on z axis in all waveguides (Top view).
The phase delay that each waveguide should introduce is represented in Eq. (1) where $\Delta \psi$ is the phase difference between adjacent waveguides, $\beta_{o}$ is the propagation constant of the rightmost waveguide, which has been used as reference, and $k_{0}$ is the propagation constant in vacuum at the operation fre-
quency. The phase delay introduced by each waveguide is properly tuned following the expression in Eq. (2) that relates the dimension $h_{y}$ with the propagation constant $\beta$ of the waveguide.

$$
\begin{gather*}
\Delta \psi=\beta_{o} t-k_{o} x_{i} \sin (\theta)+2 n \pi ; n=1,2,3 \ldots  \tag{1}\\
\beta=k_{0} \sqrt{1-\left(\frac{\pi}{k_{0} h_{y}}\right)^{2}} \tag{2}
\end{gather*}
$$

The simulation results of this new design are shown in Fig. 4

(c)

Fig. 4: Simulation results using the new design. (a) Ex-Field, (b) Hy-Field and (c) power flow.
The results are as we expected, even better than in the previous case since the edge diffraction has been minimized, although is still present.

## 4. Conclusion

To sum up, we have analysed numerically two different realizations of ENZ metamaterials constructed by stacking narrow channel waveguides. By allowing some phase progression inside each waveguide we have designed two prototypes able to convert a normally incident plane wave at the input into a plane wave with a desired output angle at the output. The prototypes presented are a prism where all waveguides have identical parameters, except the length of the waveguides and a lens where each waveguide dimension is tuned to obtain at the output the desired radiation phase pattern. In both cases, a good behaviour is observed. These results could find application in novel lenses, beam steerers, etc.

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## References

[1] M. Silveirinha and N. Engheta, Tunneling of electromagnetic energy through subwavelength channels and bends using $\varepsilon$-near-zero materials. Physical Review Letters, vol. 97, p. 157403, 2006
[2] B. Edwards, A. Alù, M.E. Young, M. Silveirinha, and N. Engheta, Experimental verification of epsilon-near-zero metamaterial coupling and energy squeezing using a microwave waveguide. Physical Review Letters, vol. 100, p. 033903, 2008.
[3] A. Alù, M. G. Silveirinha, A. Salandrino, and N. Engheta, Epsilon-near-zero metamaterials and electromagnetic sources: Tailoring the radiation phase pattern, Physical Review, vol. 75, p. 155410, 2007

