Modal Analysis of Waveguides Containing Minkowskian Isotropic Media (MIM) and the Perfect Electromagnetic Conductor (PEMC)

Filipa R. Prudêncio[1], Sérgio A. Matos[2] and Carlos R. Paiva[1]

[1]Instituto de Telecomunicações, Department of Electrical and Computer Engineering
Instituto Superior Técnico – Technical University of Lisbon
Av. Rovisco Pais, 1, 1049-001 Lisboa, Portugal
Email: filipa.prudencio@lx.it.pt, carlos.paiva@lx.it.pt
[2]Instituto de Telecomunicações, Department of Information Science and Technology
ISCTE – University Institute of Lisbon
Avenida das Forças Armadas, 1649-026 Lisboa, Portugal
Email: sergio.matos@lx.it.pt

Abstract

A perfect electromagnetic conductor (PEMC) gives rise to a new type of boundary. However, as an actual medium, the PEMC should be considered as a limit of a more general class of Tellegen media which has been called Minkowskian isotropic media (MIM). Actually, a MIM is the most general medium that is isotropic for the whole class of inertial observers, i.e., its isotropy is an observer-independent characterization in Minkowskian spacetime. The most important feature of MIM is that it enables a unique and well-defined electromagnetic field in the interior of a PEMC – as oppose to other research approaches previously published in the literature. In this work, following that new unified MIM/PEMC approach, the guided electromagnetic wave propagation in a symmetric slab containing MIM layers is developed. The main goal is to show how the special case where the MIM layers reduce to PEMC layers can be easily handled, leading to dispersion equations with nontrivial electromagnetic field profiles in the regions filled with a PEMC as the corresponding solutions.

1. Introduction

The perfect electromagnetic conductor (PEMC) was introduced by Lindell & Sihvola in [1]. This definition stems from a 4D representation using differential forms [2]. In Minkowskian spacetime the constitutive relation of these media corresponds to a simple scalar relation between the two-forms used to represent the electromagnetic fields. A fundamental implication of this definition is that a PEMC is an invariant characterization, i.e., it has the same identity for the whole class of inertial observers. The initial 3D representation of a PEMC, which has been adopted in the literature, cannot provide a unique characterization of the corresponding electromagnetic field. Nevertheless, a PEMC can be just considered as simple ideal boundary [3]-[4]: $\vec{D} - M\vec{B} = 0$ and $\vec{H} + M\vec{E} = 0$. Several attempts were made to find out what occurs inside a PEMC as a medium [5]-[6]. In [7] it is shown, using a more general class of media (MIM), how the PEMC as boundary can be defined as a real medium.

In this paper, using this new approach to address a specific canonical structure with PEMC. We analyze a slab of a conventional isotropic medium surrounded by a PEMC medium (symmetric PEMC-SIM-PEMC slab). To get this ideal structure we first analyze the realist model of these ideal case – a symmetric MIM-SIM-MIM slab. By looking to a PEMC as a limit of a MIM also provides a realist model of the ideal PEMC case, as a good conductor does for a PEC.

2. Analysis of a symmetric MIM-SIM-MIM slab

A MIM is an achiral and nonreciprocal bi-isotropic medium where the refractive index is given by $n^2 = \varepsilon\,\mu - \kappa^2 = 1$. The boundary definitions of this medium are $\vec{D} - M\,\vec{B} = (c^{-1}\,\Gamma)\,\vec{E}$ and $\vec{H} + M\,\vec{E} = (c\,\Gamma)\,\vec{B}$, where the admittance M is expressed as $M\,\eta_0 = \tan(\theta)$ and the admittance Γ is related with the parameter δ, that is, $\delta = \Gamma / M$. Therefore, the concept of a MIM can be studied only using the parameters (δ,θ). In this section a symmetric MIM-SIM-MIM slab, as depicted in Fig. 1, is analyzed. Given a symmetric structure the propagating modes can be divided in even and odd modes. Applying the boundary conditions, the even and the odd modal equation are, respectively, given by,

$$\begin{cases} 2\,\delta\left[w^2\cos^2(u)\,n_d^2 + u^2\sin^2(u)\right] - w\,u\,\sin(2\,u)\,\Delta = 0 \\ 2\,\delta\left[w^2\sin^2(u)\,n_d^2 + u^2\cos^2(u)\right] + w\,u\,\sin(2\,u)\,\Delta = 0 \end{cases}, \tag{1}$$

where $\Delta = (1+\delta^2)\tan(\theta)\,\mu_d + \cot(\theta)\,\varepsilon_d$. The refractive index of a SIM is $n_d = \sqrt{\varepsilon_d\,\mu_d}$. The normalized transverse wavenumber and the normalized transverse attenuation constant are described by $u = h\,d$ and $w = \alpha\,d$, respectively. The normalized frequency v is defined as $v = \sqrt{u^2 + w^2}$. Taking $\delta \to 0$ a PEMC is introduced as a limit of a MIM. Then, a PEMC is solely based on the parameter θ thereby reducing to a simple boundary. Therefore a symmetric PEMC-SIM-PEMC slab is studied. Now, for the even and odd modes, the modal equation is described by $\sin(u)\cos(u) = 0$. The normalized propagation constant $b\,(v) = 1 - u^2/v^2$ shows the dispersion diagrams for the even and odd modes (Figs. 2 and 3) for the MIM case ($\delta = 2$) and the PEMC case ($\delta = 0$). The surface modes are hybrid modes H_m for both structures, where the subscript m indicates that the mode is number m. Note that, the H_2 and H_3 modes have the same dispersion diagram for a PEMC-SIM-PEMC slab. However, these two modes have different magnitudes of the electromagnetic field, as shown in Figs. 4 and 5, respectively. Also, one should stress that there is a transverse electromagnetic (TEM) mode for the even modes, that is, for the solution $u = 0$. For this mode, the electromagnetic field inside the SIM region is constant.
The PEC and the PMC are particular cases of the PEMC, with $\theta = \pm\,90^\circ$ and $\theta = 0^\circ$, respectively. For the even modes one obtains a PEC or a PMC using $\sin(u) = 0$ whereas for the odd modes one needs $\cos(u) = 0$.

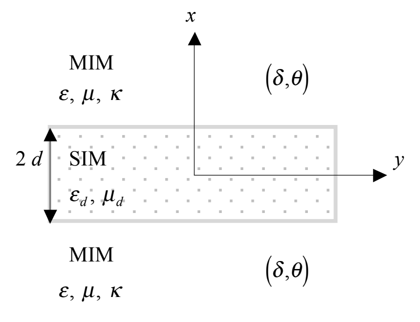

Fig. 1: Symmetric MIM-SIM-MIM slab

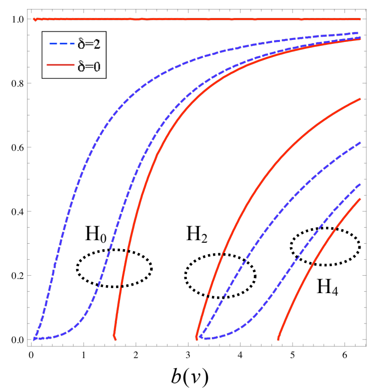

Fig. 2: Dispersion diagram for the even propagating modes, using $\theta = 45^\circ$, $\varepsilon_d = 2$ and $\mu_d = 2$.

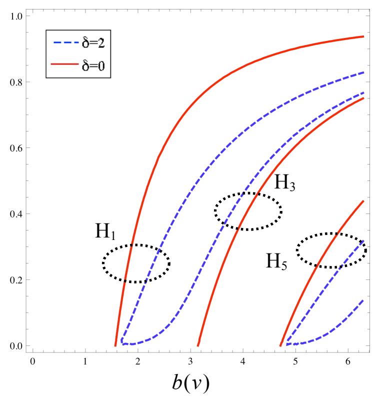

Fig. 3: Dispersion diagram for the odd propagating modes, using $\theta = 45^\circ$, $\varepsilon_d = 2$ and $\mu_d = 2$.

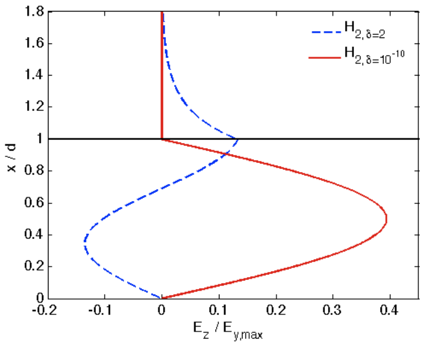

Fig. 4: The component E_z for the H_2 even mode.

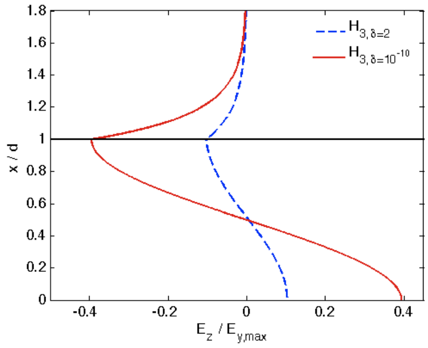

Fig. 5: The component E_z for the H_3 odd mode.

For the H_3 mode, the complex Poynting vector $\vec{S}$ is a pure imaginary vector described by $\vec{S}(x,y,z)=\left[i\,\beta A^2\,u^2\cos(u)^2\cot(\theta)/(w\,\eta_0\,k_0^2\,\mu_d^2)\right]\exp\left[-\alpha\,(x-\mathrm{d})\right]\vec{u}_y$, where A is the electric field magnitude and β corresponds to the propagation constant. Then, the instantaneous Poynting vector $\left\langle\vec{S}\right\rangle_t=\mathrm{Re}\,\left\{1/2\,(\vec{E}\times\vec{H}^*)\right\}$ vanishes inside a PEMC.

3. Conclusion

The unique way of characterizing the PEMC as a medium – not just a boundary – is to consider it as particular case of a MIM and making $\delta\rightarrow 0$.

In this paper we have shown how the electromagnetic field inside a PEMC is well-defined for a symmetric PEMC-SIM-PEMC slab. The analytical expression of the complex Poynting vector inside the PEMC is presented for the H_3 mode – although, as is well-known, the instantaneous Poynting vector vanishes inside the PEMC. Furthermore, a modal bifurcation effect is observed in the dispersion diagrams when a symmetric MIM-SIM-MIM slab is considered. When a MIM is replaced by a PEMC this effect disappears due to the vanishing of the right-hand side of the constitutive relations of a MIM, i.e., when these constitutive relations reduce to those of a PEMC. There is another interesting result: by viewing the PEMC as a limit of a MIM, one gets two distinct electromagnetic field profiles corresponding to the same dispersion diagram thereby leading to two different modes.

Acknowledgement

This work was partially founded by Fundação para a Ciência e a Tecnologia (FCT) project PEst-OE/EEI/LA0008/2011 under a Doctoral grant (SFRH/BD/67090/2009).

References

[1] I. V. Lindell and A. H. Sihvola, "Perfect electromagnetic conductor", *J. Electromagn. Waves Appl.*, vol. 19, no. 7, pp. 861-869, 2005.
[2] I. V. Lindell, *Differential Forms in Electromagnetics.* New York: Wiley/IEEE Press, 2004.
[3] I. V. Lindell and A. H. Sihvola, " Transformation method for problems involving perfect electromagnetic conductor (PEMC) structures," *IEEE Trans. Antennas Propag.,* vol. 53, no. 9, pp. 3005-3011, Sept. 2005.
[4] A. H. Sihvola and I. V. Lindell, "Perfect electromagnetic conductor medium," *Ann. Phys*. (Berlin), vol. 17, no. 9-10, pp. 787-802, 2008.
[5] I. V. Lindell and A. H. Sihvola, "Reflection and transmission of waves at the interface of the perfect electromagnetic conductor (PEMC)," *Progress in Electromagnetics Research B*, vol. 5, pp. 169-183, 2008.
[6] B. Jancewicz, "Plane electromagnetic wave in PEMC," *J. Electromagn. Waves Appl.*, vol. 20, no. 5, pp. 647-659, 2006.
[7] C. R. Paiva, S. A. Matos, "Minkowskian Isotropic Media and the Perfect Electromagnetic Conductor," *IEEE Trans. Antennas Propag.*, (accepted for publication).