

# Suppression of the Chromatic Aberrations Using a Nanowire Metamaterial

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#### Abstract

Achromatic doublets have been used for decades to minimize the effects of chromatic aberrations inherent to single-glass optical lenses. Here, we propose a fundamentally different solution to correct the chromatic aberrations based on a nanowire metamaterial with low loss anomalous dispersion. We show that by coating a standard glass lens with the metamaterial it may be possible to design a compensated bi-layer lens with nearly no chromatic aberrations. The performance of the metamaterial based lens is compared with that of an achromatic doublet, showing that it may be an interesting alternative to the conventional solution.

### **1. Introduction**

The performance of single-glass optical lenses is limited by the dispersive nature of the glass, which causes the lens to provide an imperfect focusing of the different colours of light to the same convergence point. Such a limitation known as chromatic aberration is rooted in fundamental physical restrictions related to the causality and passivity of the dielectric response. Causality and passivity determine that the index of refraction of any conventional low loss dielectric material is a strictly increasing function of frequency [1]. Let us consider an optical lens formed by two thin refracting surfaces standing in air, with radius of curvature  $R_1$  and  $R_2$  and index of refraction  $n_1$  and  $n_2$  (upper-half panel of Fig. 1a). In this scenario, the focal length f of the optical system satisfies the well known Lensmaker's equation:

$$\frac{1}{f} = (n_1 - 1)\frac{1}{R_1} + (n_2 - 1)\frac{1}{R_2}.$$
(1)

If the optical lens is formed by a single glass i.e.,  $n_1 = n_2 = n_g(\omega)$ , it is clear from Eq. (1) that the focal length of the system f decreases with the frequency because the index of refraction  $n_g(\omega)$  of glass is an increasing function of frequency. It is possible to derive from Eq. (1) that the focal length can remain invariant with respect to changes in the frequency if the following condition is satisfied:

$$\dot{n}_1/\dot{n}_2 = -(R_1/R_2),$$
 (2)

where  $\dot{n}_i = dn_i / d\omega$  (*i*=1,2). Assuming positive and negative values of  $R_i$  for convex and concave surfaces (as seen from the air region), respectively, and that the lens is composed by two thin refracting glasses, it is clear that in order to satisfy Eq. (2), i.e. to suppress the chromatic aberrations, the optical system must be composed by a concave refracting glass paired with a convex one, since  $\dot{n}_1$  and  $\dot{n}_2$  must have the same signs. In fact, this solution to minimize the chromatic aberrations has been used for decades and the resulting optical device is called an achromatic doublet (upper-half panel of Fig. 1a) [2,3]. Achromatic doublets permit suppressing the chromatic aberrations of two wavelengths i.e., two colours, and are usually composed by a convex Crown glass lens paired with a concave Flint glass lens, where the designations of Flint or Crown stem from the higher or lower dispersive nature of the glass, respectively. Now let us assume that the thin lens is instead formed by two convex refracting surfaces, i.e.  $R_1$  and  $R_2$  are positive. In such a scenario, it is evident that in order to satisfy the condition (2) a material with anomalous dispersion is required, because  $\dot{n}_1$  and  $\dot{n}_2$  must have opposite signs. As



far as we know, this solution has not been considered before because regimes of anomalous dispersion in conventional materials imply very significant loss. In this paper we show how this limitation can be overcome by using a nanowire based metamaterial characterized by a low loss broadband index of refraction with anomalous dispersion in the optical regime. We show with geometrical optics and fullwave simulations that by coating a standard plano-convex Flint glass lens with the nanowire based metamaterial (lower-half section of Fig. 1a) it is possible to design a bi-layer *compensated* lens that nearly corrects the chromatic aberrations of all colours of light.

#### 2. Bi-layer compensated lens

The metamaterial analysed here is a double wire medium composed by long thin metallic wires. This microstructured material may have interesting applications such as the realization of ultra-compact waveguides [4], superlenses [5], amongst others. Particularly, it has been shown recently that the double wire medium may enable the realization of a dielectric response with a low loss broadband anomalous dispersion [6]. This is possible because the metamaterial is strongly spatially dispersive, and the restrictions on the dispersion of the index of refraction for this class of media are less strict than for conventional dielectrics. This material is formed by two nonconnected arrays of metallic nanowires with radius  $r_w$  and relative permittivity  $\varepsilon_m$ , such that each array of parallel wires is tilted by  $\pm \theta_0^{\circ}/2$  with respect to the x axis. The two arrays of wires lie in planes parallel to the xoz plane and the distance between adjacent wires is a / 2 [Figs. 1b and 1c]. For simplicity of modeling, and without loss of generality, it is assumed that the wires stand in air.



Fig. 1: (a) The upper-half section shows a standard achromatic doublet whereas the lower-half section shows a biconvex metamaterial based lens that also reduces the chromatic aberrations. Panels (b) and (c) show cuts of a "double wire medium" slab with thickness L, along the *xoy* and *xoz* planes, respectively. (d) focal plane curve calculated using geometrical optics [Eq. 1]. The dark yellow curve and the blue curve represent the foci of the achromatic metamaterial biconvex lens and of the conventional achromatic doublet, respectively.

In order to compare the performance of the proposed metamaterial based lens against that of a conventional achromatic doublet, we assume that the required focal length of both lenses is  $f \sim 7.8 \,\mu\text{m}$  at the wavelength of the *d* Fraunhofer spectral line i.e., at  $\lambda_d \approx 0.587 \,\mu\text{m}$ . Our compensated lens is composed by a plano-convex glass lens with refractive index  $n_2 = n_{SF10}$ , coated by a plano-convex double wire medium lens with effective refractive index  $n_1 = n_{xw}$  (lower-half lens of Fig. 1a). The metamaterial is formed by Al wires, with diameter  $2r_w = 24nm$ , lattice constant a = 110nm and the wires are tilted so that  $\theta_0 = 168^\circ$ . The dielectric material is dense flint glass SF10 and the refractive indices of the metal and SF10 are calculated using the parameters reported in [7] and [8], respectively. Based on a generalization of the effective medium model of [6], we can estimate  $\dot{n}_{xw} = -0.025 \times 10^{-3} [1/THz]$  and  $\dot{n}_{SF10} = 0.15 \times 10^{-3} [1/THz]$  and hence from (1) and (2) the radii of curvature of the two surfaces are  $R_1 = 8.4 \,\mu\text{m}$  and  $R_2 = 49.6 \,\mu\text{m}$ . The achromatic doublet is formed by a plano-convex Crown glass N-BK7 lens with refractive index  $n_2 = n_{BK7}$  [8] coated by a plano-concave dense flint glass SF10 lens  $(n_1 = n_{SF10})$ . Since  $\dot{n}_{BK7} = 0.047 \times 10^{-3} [1/THz]$ , from (1) and (2) the radii of curvature of the two interfaces of the doublet are  $R_1 = -6.9 \,\mu\text{m}$  and  $R_2 = 2.2 \,\mu\text{m}$ . In Fig. 1d we depict the focal curve (using Eq.



[1]), i.e., the change  $\delta f$  in the focal length caused by a change  $\Delta n$  in the refractive index of both materials, as a function of the wavelength. The dark yellow and blue curves refer to the compensated lens and the standard doublet lens, respectively. As seen in Fig. 1d, our lens has a performance similar to the conventional doublet, and thus it may be an interesting alternative to the conventional solution.

## **3.** Full-wave simulations

In order to further validate the proposed solution to reduce the chromatic aberrations, we calculated with full wave simulations the focal curve of the metamaterial lens. For simplicity of numerical modelling, in this case the metamaterial is such that  $2r_w = 30nm$ , a = 100nm and  $\theta_0 = 90^\circ$ . The optical lens is designed so that  $f \sim 7.8 \,\mu\text{m}$  at  $\lambda_0 = 0.56 \,\mu\text{m}$ . Similar to the previous approach, from (1) and (2) it is possible to calculate that the radii of the two surfaces,  $R_1 = 46.24 \,\mu\text{m}$  and  $R_2 = 9.25 \,\mu\text{m}$ . The central thicknesses of the two refractive surfaces are taken equal to  $d_1 = 0.26 \,\mu m$  and  $d_2 = 1.31 \,\mu m$ , respectively. The performance of a plano-convex lens with the same focal power and made exclusively by dense Flint SF10 glass with  $R_{1,s} = \infty$ ,  $R_{2,s} = 6.1 \,\mu m$ ,  $d_{2,s} = 1.31 \,\mu m$  and  $n_2 = n_{\text{SF10}}$  is also analysed.



0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75

Fig. 2: Focal plane curve as a function of the wavelength. The discrete stars (FDFD) and the discrete circles (CST Microwave Studio) represent the position of the foci of the achromatic biconvex metamaterial lens. The solid curve (FDFD) represents the focal curve of a thin plano-convex glass lens.

In Fig. 2 we show the focal curve of the compensated lens obtained using a full wave simulator [9] that takes into account the metal loss and all the minute details of the double wire medium (yellow circles), and that obtained using a finite-differences frequency-domain full wave simulator based on the effective medium theory (yellow stars). The agreement between both simulations is quite good. The blue curve represents the focal curve of the single glass lens, confirming that that the proposed design performs far better than an ordinary glass lens, and may possibly be an exciting alternative to the conventional achromatic doublet. At the conference, we will also discuss the possibility of suppressing the chromatic aberrations with a single-material lens with engineered flat dispersion.

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