Design and modeling of Luneburg lens based on dodecagonal photonic quasicrystal

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Abstract

We introduce graded photonic quasicrystals (GPQs) and investigate properties of such structures on the example of Luneburg lens based on a dodecagonal photonic quasicrystal. It is shown that the graded photonic quasicrystal lens has better focusing properties as compared with the graded photonic crystal lens in a frequency range suitable for experimental realization. The proposed graded photonic quasicrystals can be used in optical systems where compact and powerful focusing elements are required.

1. Introduction

Gradient index (GRIN) structures have received increased attention in the recent literature from invisibility cloaks [1] to planar devices such as Luneburg lens [2]. Recently, the concept of graded photonic crystals (GPCs) was introduced [3], where it was shown how to bend a path of light by two-dimensional (2D) GPC with one-dimensional lattice gradient. The possible realization of isotropic GRIN media on the use of 2D GPCs in metamaterial regime at optical frequencies was considered [4]. The previous studies on 2D GPCs were focused on realization of particular functions, but they did not consider a determination of the lattice geometry which would be producing the preferable GRIN structures. In all the earlier works a square or hexagonal lattice was used for implementation of GPCs. Photonic quasicrystals (PQs) have a high rotational symmetry, therefore their optical properties can be almost isotropic [5] and such structures are more preferable for the lower refractive index materials.

2. Results

We consider the photonic crystal with a square lattice and the PQ with a square-triangle 12-fold-symmetric tiling. This PQ possesses a 12-fold symmetry and it is called a dodecagonal quasicrystal[6]. From the Maxwell-Garnett effective medium theory, the effective permittivity of GPCs and GPQs with rods embedded in a background material can be written as [7]:

$$\varepsilon_{plane} = \varepsilon_{host} + \frac{f\varepsilon_{host}(\varepsilon_{rods} - \varepsilon_{host})}{\varepsilon_{host} + 0.5(1 - f(\varepsilon_{rods} - \varepsilon_{host})} \tag{1}$$

while ε_z is obtained using the formula:

$$\varepsilon_z = (1 - f)\varepsilon_{host} + f\varepsilon_{rods} \tag{2}$$

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where f – the filling fraction of rods, ε_{host} – the permittivity of the host medium, ε_{rods} – the permittivity of dielectric rods. The effective refractive indices are $n_{te} = \sqrt{\varepsilon_{plane}}$ and $n_{tm} = \sqrt{\varepsilon_z}$. For fixed value of rod radius r the filling fraction of rods can be found by the formula $f = N\pi r^2/S$, where S– the area of photonic structures, N – the number of rods. This formula is applicable if the rods are uniformly distributed in the structure that is performed for the case square-triangle tiling. Finally, in the case of TE-polarization, the radius r_{te} is obtained from Eq. (1):

$$r_{te} = \sqrt{\frac{S(\varepsilon_{host} - n^2)(\varepsilon_{host} + \varepsilon_{rods})}{\pi N(\varepsilon_{host} + n^2)(\varepsilon_{host} - \varepsilon_{rods})}} \tag{3}$$

for TM-polarization the radius r_{tm} is obtained from Eq. (2):

$$r_{tm} = \sqrt{\frac{S(\varepsilon_{host} - n^2)}{\pi N(\varepsilon_{host} - \varepsilon_{rods})}} \tag{4}$$

Analytical Eq. (3) and (4) allow us to design planar GRIN devices based on PQs. Next, we apply of these equations for design of GPQ Luneburg lens for TE-polarization.

The Luneburg lens [8] is a GRIN element with refractive index distribution $n(\rho) = n_0\sqrt{2 - (\rho/R)^2}$, where n_0 – the refractive index outside the lens region, ρ – the radial polar coordinate within the lens region and R – the radius of the lens. We will investigate a model case, when the Luneberg lens is placed in the air ($n_0 = 1$). For realization of this lens we use the PC with the square lattice as it was done in [4] and the dodecagonal PQ based on SiO_2 dielectric rods (ε=2.37) in air with different radii. Fig. 1 shows the implementation of the Luneburg lens based on the dodecagonal PQ (see Fig. 1(a)) and the PC with the square lattice (see Fig. 1(b)). The distance between the rods is a and the radius of the lenses is $R = 15a$.

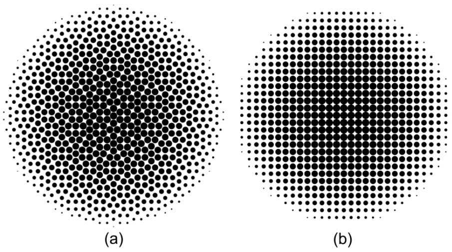

Fig. 1: A schematic of Luneburg lens based on the dodecagonal PQ (a) and the square lattice (b) with the graded radii of the rods.

In order to evaluate the lens properties, we calculated the transmission coefficients for the TE-polarization plane wave as a function of the frequency ω. We performed simulations using the finite-difference time-domain method (FDTD), implemented as a freely available software package [9], with a resolution of 32 pixels per lattice spacing.

Fig. 2 shows the transmission coefficients for the GPC and GPQ lenses, and also for the original Luneburg lens for TE-polarization. All three lenses show a similar performance in the frequency range 0.1–0.25 ($\omega a/2\pi c$). Further increasing of the frequency ω above 0.32 leads to the strong reflection from the GPC lens, while the reflection from the GPQ lens is not significantly increased. The increase in the reflection from the GPC lens can be explained by the fact that the incident electromagnetic plane wave does not perceive lens structure as a locally homogeneous medium further more. For the experimental realization of the GRIN devices it is desirable that a high efficiency was achieved at high frequencies. The size of the minimal elements of the device increases in this case, what is important because of the limited resolution of the e-beam lithography. The Fig. 2 illustrates that the GPQ lens has a much greater efficiency in the frequencies range 0.34–0.38 ($\omega a/2\pi c$) than the GPC lens. Let us compare the focusing properties of the two lenses at the frequency ω=0.376.

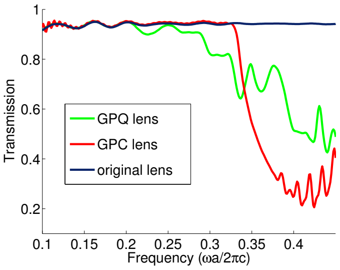

Fig. 2: (Color online) The transmission coefficients for the GPQ lens (green), the GPC lens (red) and the original Luneburg lens (blue) for TE-polarization.

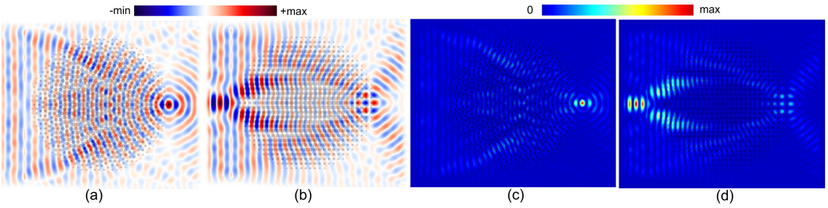

Fig. 3: (Color online) The distributions of H_z field for the GPQ lens (a) and the GPC lens (b). The magnetic field intensity distributions for the GPQ lens (c) and the GPC lens (d).

Fig. 3(a) and 3(b) describe the distributions of the H_z field (ω=0.376) for the GPQ lens and the GPC lens, respectively. Fig. 3(c) and 3(d) show the magnetic field intensity distributions for the GPQ lens and the GPC lens, respectively. As can be seen the GPQ lens works better as compared with the GPC lens, the TE-polarization plane wave from the left side is focused onto the opposite right side. We observe strong Bragg reflection in the center of the GPC lens (see Fig. 3(d)), so the performance of the lens is very low. High efficiency of the GPQ lens is determined by the high symmetry of the quasicrystalline structure and absence of periodicity, in consequence of which the Bragg reflection condition is not satisfied at a given frequency.

3. Conclusion

The Luneburg lens based on the dodecagonal photonic quasicrystal with graded radii of the rods has been investigated. It has been shown that the GPQ lens has better focusing properties than the GPC lens in the frequency range 0.34–0.38 ($\omega a/2\pi c$) suitable for experimental realization. We believe that GPQ structures can be used in prospective integrated photonic devices.

References

[1] J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, Nat. Mater., vol. 8, p. 568, 2009.
[2] J. Hunt, T. Tyler, S. Dhar, Y.-J. Tsai, P. Bowen, S. Larouche, N. M. Jokerst, and D. R. Smith, Opt. Express, vol. 20, p. 1706, 2012.
[3] E. Centeno and D. Cassagne, Opt. Lett., vol. 30, p. 2278, 2005.
[4] B. Vasić, G. Isić, R. Gajić and K. Hingerl, Opt. Express, vol. 18, p. 20321, 2010.
[5] P. N. Dyachenko, N. D. Kundikova, and Y. V. Miklyaev, Phys. Rev. B, vol. 79, p. 233102, 2009.
[6] M. Oxborrow and C. L. Henley, Phys. Rev. B, vol. 48, p. 6966, 1993.
[7] A. Sihvola, Electromagnetic Mixing Formulas and Applications, IEE Publishing, 1999.
[8] R. K. Luneburg, Mathematical Theory of Optics, Brown U. P., 1944.
[9] A. F. Oskooi, D. Roundy, M. Ibanescu, P. Bermel, J. D. Joannopoulos, and S. G. Johnson, Comput. Phys. Commun., vol. 181, p. 687, 2010.