Perfect imaging of point sources with positive refraction

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Abstract

The capability of a device called the Spherical Geodesic Waveguide (SGW) to produce images with details below the classic Abbe diffraction limit (super-resolution) is analyzed here. The SGW is an optical system equivalent (by means of Transformation Optics) to the Maxwell Fish Eye (MFE) refractive index distribution. Recently, it has been claimed that the necessary condition to get super-resolution in the MFE and the SGW is the use of a Perfect Point Drain (PPD). The PPD is a punctual receptor placed in the focal point that absorbs the incident wave, without reflection or scattering. A microwave circuit comprising three elements, the SGW, the source and the drain (two coaxial lines loaded with specific impedances) is designed and simulated in COMSOL. The super-resolution properties have been analyzed for different position of the source and drain and for two different load impedances: the PPD and the characteristic line impedance. The results show that in both cases super-resolution occurs only for discrete number of frequencies. Out of these frequencies, the SGW does not show SR in the analysis carried out.

1. Introduction

Leonhardt [1] analyzed Helmholtz wave fields in the MFE lens in two dimensions (2D). These Helmholtz wave fields describe TE-polarized modes in a cylindrical MFE, i.e., modes in which electric field vector points orthogonally to the cross section of the cylinder. Leonhardt found a family of Helmholtz wave fields which have a monopole asymptotic behavior at an object point as well as at its stigmatic image point. Each one of these solutions describes a wave propagating from the object point to the image point. This wave coincides asymptotically with an outward (monopole) Helmholtz wave at the object point, as generated by a point source, and with an inward (monopole) wave at the image point, as it was sunk by an "infinitely-well localized drain" (which we call a "perfect point drain", PPD). In ref. [2] we proved that such PPD can be modelled as a small dissipative region with a definite complex permittivity which depends on its size as well as on the frequency. This PPD absorbs the incident wave, with no reflection or scattering. All these results can be translated to the SGW as can be demonstrated with Transformation Optics [3]. Leonhardt assumed that the ability of the MFE to propagate the wave, generated by a point source, toward to a PPD was enough to guarantee perfect imaging. This does not seem to be sufficient, since it does not provide information on how much power the PPD will absorb when it is displaced out of the image point. One experiment was recently carried out to support the super-resolution (SR) capability in the MFE for one microwave frequency [4]. Its results showed that two sources with a distance of $\lambda/5$ from each other (where λ denotes the local wavelength $\lambda = \lambda_0/n$) could be resolved with an array made up of 10 drains spaced $\lambda/20$, which exceeded the $\sim\lambda/2.5$ classic diffraction limit. In this experiment the receptor is not the PPD but only a coaxial probe loaded with its characteristics impedance. Here we analyze the SGW to clarify the role of the PPD for a broader frequency band. We have obtained two noticeable results: SR occurs only in the neighborhood of a specific set of frequencies (called notch frequencies, and located near the Schumann frequencies), and it happens not only for the PPD but also when the line is loaded with its characteristic impedance which emulates a non-perfect drain. The different drains affect to the bandwidth for which the SR is achieved. This bandwidth is 20 times higher with PPD than in other drain case analyzed.

2. Model description.

The SGW is bounded by two metallic spherical shells. The refractive index media between shells is graded proportional to the law $1/\rho$, where ρ is the radial coordinate. In our examples, the spherical shells are close compared to their size and using air as the media in between shells gives the same results as the exact graded index SGW prescription. Two coaxial lines are used to simulate the point source and the drain. The microwave circuit consists of the generator V_g with impedance Z_g (on the source side), coaxial lines, the SGW, and the load with impedance Z_L (on the drain side) (Fig. 1). The SGW dimensions are $R_M = 1005$ mm, $R_m = 1000$ mm, $a = 5$ mm (diameter of the inner coaxial conductor), b=10 mm (inner diameter of the outer coaxial conductor), L = 20 mm.
The circuit has been analyzed when Z_L=Z_g=Z_0, where Z_0 is the characteristics impedance of the coaxial line, (Z_0=41.54Ω in our example [5]) and when Z_L=Z_g=Z_{ppd}, where Z_{ppd} is the load that does not produce reflection in the SGW, i.e., the load that makes the coaxial line work as a perfect drain [6] , which for our example is the one shown in Fig. 1 right.

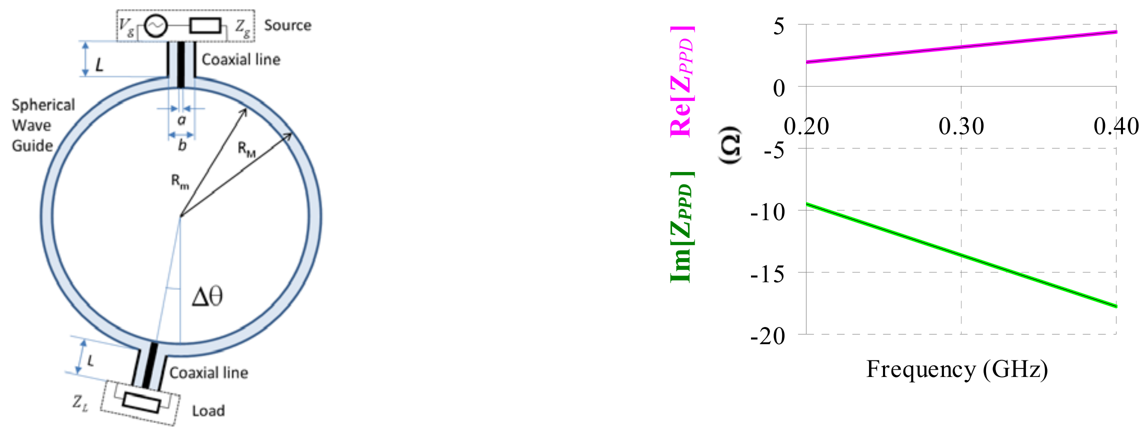

Fig. 1. (Left) Microwave circuit is formed by: the source (V_g and Z_g) connected to a coaxial transmission line of length L, the load connected (Z_L) to other identical transmission line and the spherical waveguide. R_M and R_m are radius of the external and internal metallic spheres. (Right) Real and imaginary parts of Z_{ppd} .

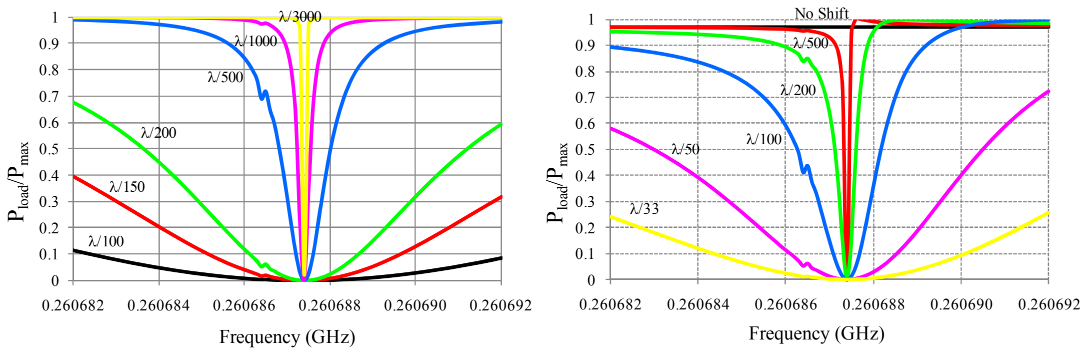

Fig. 2. P_{load}/P_{max} as function of the frequency in a narrow band around a SGW frequency for different drain port positions. (Left) Z_L=Z_g=Z_0, (Right) Z_L=Z_g=Z_{ppd}. The label on each curve indicates the distance between the drain and the source antipode.

3. Simulation results.

Fig. 2 shows P_{load}/P_{max} (P_{load} is the power delivered to the load Z_L and P_{max} is the maximum power deliverable by the generator P_{max}= $|V_g|^2/(4\mathrm{Re}(Z_g))$ as a function of the frequency and for different drain coaxial line positions (the label on each curve indicates the distance between the center of the drain port and the source port antipode). The shifts are in all cases much smaller than wavelength (from λ/33 to λ/500 for $Z_L=Z_g=Z_0$ and λ/140 to λ/3000 for $Z_L=Z_g=Z_{ppd}$, where λ=1.15084047 m is the wavelength corresponding to f=0.2606873 GHz).

These results are quite surprising, since close to specific frequencies the power transmitted to the drain suddenly reduces to a value near zero. Fig. 3 shows the bandwidth vs. N, meaning that the resolution is better that λ /N. The bandwidth corresponding to the point λ /N of this curve is calculated from the curve labeled λ /N in Fig. 2 as $f_{max}-f_{min}$ where f_{max} and f_{min} are the frequencies fulfilling P_{load}/P_{max}=0.1. The linear dependence shown in Fig. 3 (slope -2) reveals that the product N^2×bandwidth is constant in the range analyzed here**.**

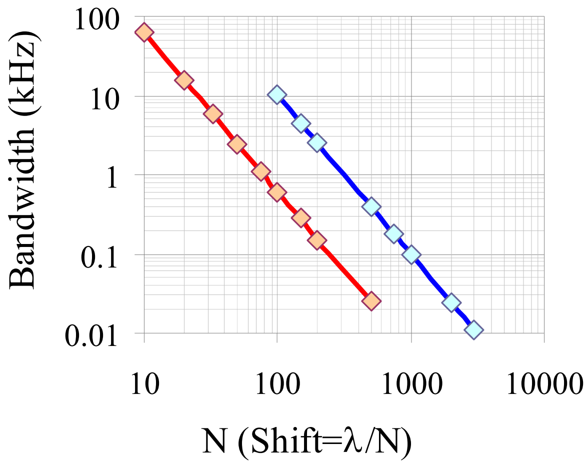

Fig. 3. Bandwidth as a function of the resolution. The *x* axis shows N, meaning that the resolution is better than λ /N. The red line corresponds to a load $Z_L=Z_g=Z_0$, and the blue line to $Z_L=Z_g=Z_{ppd}$ (PPD). Note that using the PPD load the bandwidth increases 20 times.

4. Discussion

Leonhardt in [1] proved that a point source is perfectly imaged in the MFE at any frequency if a mathematical concept called Perfect Drain is used. In [2] we showed that this concept can be implemented with a dissipative region of a definite dielectric constant. The experiments in [4] have shown super-resolution properties of the MFE at a single frequency. In this experiment, the coaxial probes were loaded with their characteristic impedances, so the absorption of the incident wave was not perfect. The simulations presented here show that super-resolution only occurs for a particular set of frequencies known as notch frequencies and it happens for the two types of loads analysed. We have shown that source displacements of λ/3000 can be resolved for the system loaded with the perfect drain ($Z_L=Z_g=Z_{ppd}$), and λ/500 for the system loaded with characteristic impedance ($Z_L=Z_g=Z_0$). The frequency bandwidth for which super-resolution appears is very narrow and depends on the load used. It is 20 times wider for the perfect drain case than the other case.
Imaging a field distribution different from a point source is not straightforward from the preceding results. Consider for example two point sources. This example cannot be treated as a linear superposition of two single point-source cases because the need of a perfect drain makes the media different. Draining the incoming power when it has reached the target seems to be important to avoid interference of the reflected waves with the incoming ones. Nevertheless, the preceding results are not conclusive about the importance of this absorption for image formation.

References

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