Inhomogeneous waves of light in isotropic metamaterials

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Abstract

We theoretically investigate the propagation of inhomogeneous waves of light in metamaterials. Our results show that to obtain the true refractive index and extinction coefficient of light transmitted to metamaterial one needs to take into account inhomogeneity of the transmitted wave. Also we show that presence of negative refraction at the interface with metamaterial depends not only on the properties of metamaterial itself but also on the geometry of a problem.

1. Introduction

Propagation of light in absorptive media is very different from that in absorption-free media. In absorptive media light propagates in the form of inhomogeneous waves — waves for which planes of the constant phase and amplitude are not parallel to each other [1]. Since metamaterials are highly absorptive media, one can assume that light waves in them are also inhomogeneous.

In this work we apply the theory of inhomogeneous waves to the case of light propagation in isotropic metamaterial. We show that, as in any isotropic absorptive medium, the refractive index of light inside metamaterial is not defined only by the metamaterial properties but also by the geometry of the incident light. Our results imply that the effect of negative refraction at the interface with metamaterial can be reached not only by the negative permittivity and permeability but also by the proper choice of the geometry.

2. Results and Discussion

Subwavelength dimensions of metamaterial structure elements makes it possible to consider whole metamaterial as a bulk medium with some effective complex permittivity $\varepsilon = \varepsilon' - i\varepsilon''$ and permeability $\mu = \mu' - i\mu''$. Therefore one can consider propagation of light in metamaterial in terms of plane monochromatic waves. Electric field $\vec{E}$ of plane monochromatic wave in a medium with complex ε and μ is

$$\vec{E} = \vec{e}e^{i(\omega t - \vec{k}\cdot\vec{r})}, \tag{1}$$

where $\vec{e}$ — complex amplitude, ω — field frequency, t — time, $\vec{r}$ — radius-vector, and $\vec{k} = \vec{k}' - i\vec{k}''$ is a complex wave vector. A traditional approach considers real wave vector $\vec{k}'$ parallel to a vector of extinction $\vec{k}''$, which corresponds to a usual dumped wave. But in an absorbing medium this is only a special case. In general case Eq. (1) describes an inhomogeneous wave — the wave for which the wave vector $\vec{k}'$ is not parallel to a vector of extinction $\vec{k}''$.

To show the difference between the damped waves and inhomogeneous waves let us consider the dispersion relation for the complex wave vector $\vec{k}$:

$$\vec{k}^2 = \varepsilon_0\mu_0\varepsilon\mu\omega^2 = n^2\frac{\omega^2}{c_0^2}, \tag{2}$$

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where $c_0 = 1/\sqrt{\varepsilon_0\mu_0}$ is a velocity of light in vacuum, and the complex value $n = n' - in''$ is defined as $n^2 = \varepsilon\mu$. Next we define the refractive index m' through the length k' of the wave vector as $k' = m'\frac{\omega^2}{c_0^2}$, and the extinction coefficient m'' through the length k'' of the extinction vector as $k'' = m''\frac{\omega^2}{c_0^2}$. Substitution of these expressions to the dispersion relation (2) leads to so-called Ketteler's formulas [1]:

$$n'^2 - n''^2 = m'^2 - m''^2, \tag{3a}$$

$$n'n'' = m'm'' \cos\theta_\times, \tag{3b}$$

where $\theta_\times$ is the angle between vectors $\vec{k}'$ and $\vec{k}''$. In accordance with Eqs. (3) the real and imaginary parts of n value correspond to the refractive index and extinction coefficient only in the case of $\theta_\times = 0$ ($\vec{k}'$ and $\vec{k}''$ are parallel and codirectional). In other words, for the case of inhomogeneous waves ($\theta_\times \neq 0$), value n defined through the permittivity ε and permeability μ does not contain information about the velocity of light and rate of its absorption.

Thus knowledge of angle $\theta_\times$ allows one to find the true values of refraction index m' and extinction coefficient m''. This angle can be found if one will consider the formation of inhomogeneous wave at the interface with a metamaterial sample. In optics the traditional approach to the interface problem is a coordinate approach, when some fixed coordinate system is introduced. But in case of inhomogeneous waves this approach is not very convenient. Usually one of the axes of the introduced coordinate system is directed along the direction of light propagation. But in case of inhomogeneous wave the direction of light propagation is defined by the two noncollinear vectors $\vec{k}'$ and $\vec{k}''$. Thus there is no unique choice of the coordinate system for the interface problem. To overcome this complication we use the coordinate-free approach [2], where no coordinate system is introduced at all and all calculations are made with vectors directly.

In the coordinate-free approach the complex wave vectors of incident ("i"), reflected ("r"), and transmitted ("t") waves at the interface with two arbitrary (absorbing or not) media can be written as

$$\vec{k}_i = \vec{b} + \eta_i\hat{q}, \tag{4a}$$

$$\vec{k}_r = \vec{b} + \eta_r\hat{q}, \qquad \eta_r = -\eta_i, \tag{4b}$$

$$\vec{k}_t = \vec{b} + \eta_t\hat{q}, \qquad \eta_t^2 = \vec{k}_t^2 - \vec{a}^2, \tag{4c}$$

where $\vec{a} = [\vec{k}_i \times \hat{q}]$, $\vec{b} = [\vec{q} \times \vec{a}]$, and $\eta_\alpha = \vec{k}_\alpha \cdot \hat{q}$ with $\alpha = i, r, t$. Here $\hat{q}$ is real unit normal to the interface directed to the second medium. All other vectors and values in Eqs. (4) are complex: $\vec{a} = \vec{a}' - i\vec{a}''$, $\vec{b} = \vec{b}' - i\vec{b}''$ and $\eta_\alpha = \eta_\alpha' - i\eta_\alpha''$. The real part of Eqs. (4) corresponds to a set of real wave vectors $\vec{k}_\alpha'$ and imaginary to a set of extinction vectors $\vec{k}_\alpha''$.

Consider, for example, a set of real wave vectors $\vec{k}_i'$, $\vec{k}_r'$, $\vec{k}_t'$ (see Fig. 1). Then Eqs. (4) states that (1) all the real wave vectors of the incident, reflected, and transmitted waves lie on the plane of incidence which is defined by vectors $\vec{b}'$ and $\hat{q}$ (or $\vec{k}_i$ and $\vec{q}$); (2) the projections of the wave vectors $\vec{k}_i'$, $\vec{k}_r'$, $\vec{k}_t'$ on the interface are equal to the same constant vector $\vec{b}'$; and (3) the projections of $\vec{k}_i'$, $\vec{k}_r'$, $\vec{k}_t'$ on the unit normal to the interface are, respectively, η_i', η_r', η_t'. In other words the tangential components of real wave vectors are equal and the only difference is in their normal components. The same interpretation is true for the imaginary part of Eqs. (4) and a set of extinction vectors $\vec{k}_i''$, $\vec{k}_r''$, $\vec{k}_t''$.

We calculated the normal components of the transmitted wave vector $\vec{k}_t'$ and vector of extinction $\vec{k}_t''$ with the assumption that they can be negative (which imply negative refraction). As a result we obtained the following expressions for η_t for the absorbing medium:

$$\eta_t' = \mathrm{sign}\{\xi''\}\sqrt{\frac{|\xi| + \xi'}{2}}, \qquad \eta_t'' = \sqrt{\frac{|\xi| - \xi'}{2}}, \tag{5}$$

where $\xi = \xi' - i\xi''$ and

$$\xi' = (\varepsilon_2'\mu_2' - \varepsilon_2''\mu_2'')\frac{\omega^2}{c_0^2} - (\vec{a}'^2 - \vec{a}''^2), \qquad \xi'' = (\varepsilon_2'\mu_2'' + \mu_2'\varepsilon_2'')\frac{\omega^2}{c_0^2} - 2(\vec{a}' \cdot \vec{a}''). \tag{6}$$

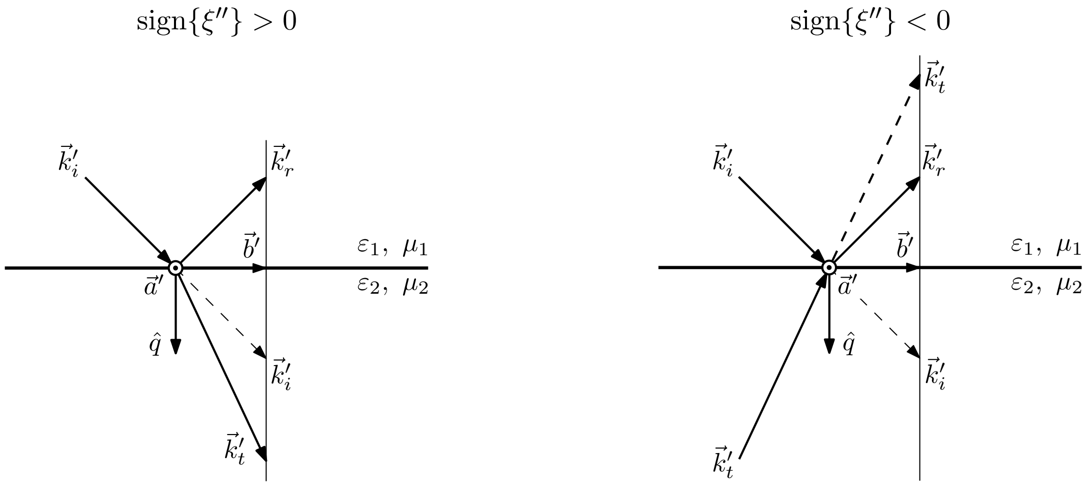

Fig. 1: Real wave vectors in case of positive (left) and negative (right) refraction.

Eq. (5) shows that the normal component η_t'' of the transmitted extinction vector $\vec{k}_t''$ is always positive. I.e. the extinction vector $\vec{k}_t''$ is directed away from the interface, which is the condition of a passive medium. But the sign of the normal component η_t' of the transmitted real wave vector $\vec{k}_t'$ depends on a value of ξ''. When $\xi'' > 0$ the refraction is positive and when $\xi'' < 0$ the refraction is negative. Eq. (6) shows that ξ'' consists of two terms. When transmitted wave is homogeneous second term is zero: $\vec{a}' \cdot \vec{a}'' = 0$. In this case the refraction is negative if $\varepsilon_2'\mu_2'' + \mu_2'\varepsilon_2'' < 0$. This is the well-known condition for the negative refraction, which imply negative ε' and μ' [3]. But in case of inhomogeneous waves the second term in ξ'' can be nonzero. This term is proportional to the scalar product of vectors $\vec{a}' = [\vec{k}_i' \times \hat{q}]$ and $\vec{a}'' = [\vec{k}_i'' \times \hat{q}]$. Vectors $\vec{a}'$ and $\vec{a}''$ specify normals to the incident planes of real wave vectors and vectors of extinction respectively. Thus condition $\vec{a}' \cdot \vec{a}'' \neq 0$ imply that these planes of incidence are not parallel. *Therefore inhomogeneity of the waves alters the traditional criterion for negative refraction.* Negative refraction is not only the consequence of material properties (negative ε' and μ') but also of geometry of the interface problem. In addition, in principle, the existence of geometrical term in Eq. (6) for ξ'' imply that the negative refraction can be observed in the medium with positive ε' and μ' or even in an absorption-free medium when the material term $\varepsilon_2'\mu_2'' + \mu_2'\varepsilon_2''$ is equal to zero. In our presentation we will consider in details the influence of problem geometry on the negative refraction and propose several geometries which allow one to observe the effects described above.

4. Conclusion

We theoretically studied the formation and propagation of inhomogeneous waves of light in isotropic metamaterial. To simplify our study we used the coordinate-free approach. Our results show that the refractive index of light and its extinction inside metamaterial are not defined only by the metamaterial properties but also by the geometry of incident light. We have shown that this geometry dependence leads to the modified criterion of the negative refraction, which allows one to observe the negative refraction in a medium with positive permittivity and permeability. We hope that our findings will help one to optimize the design of optical elements based on metamaterials such as superlenses.

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