Lossy Wave Transmission Through Graded Interfaces Between RHM and LHM Media - Case of different Loss Factors in the two Media

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Abstract

The transmission and reflection properties of lossy structures involving left-handed materials with graded permittivity and permeability have been investigated. We present an exact analytical solution to Helmholtz' equation for a lossy case with the graded both real and imaginary parts of permittivity and permeability profile changing according to a hyperbolic tangent function along the direction of propagation. This allows for different loss factors in the two media. The expressions and graphical results for the field intensity along the graded structure are presented.

1. Introduction

We present an exact analytical solution of Helmholtz' equation for the propagation of electromagnetic waves through a lossy graded metamaterial structure, where both the permittivity and the permeability vary according to a hyperbolic tangent function. Such structures, with neglected losses, were studied in the framework of metamaterial gradient index lenses in [1], showing that they provides an additional degree of freedom that can be used to reduce geometrical aberrations. A gradient metamaterial lens was also demonstrated experimentally by Smith [2]. Theoretical investigations of graded structures including LHM Media have been done only in the recent few years [3, 4, 5, 6]. These theoretical investigations have so far only covered cases with neglected losses or uniform losses with same loss factors in both RHM and LHM media. In the present paper we offer the most general case of lossy wave propagation with constant impedance throughout the entire structure, where loss factors can be chosen arbitrarily in both RHM and LHM media. This provides the opportunity to model the significantly higher losses in LHM materials compared to those in the RHM materials.

2. Field equations

We assume the time-harmonic waves with an $\exp(-i\omega t)$ dependency in isotropic materials, where the effective medium approximation is valid. The geometry of the problem is illustrated in Fig. 1. The electric field is directed along the y-axis, $\vec{E}(\vec{r}) = E(x)\vec{e}_{\mathrm{y}}$, whereas the magnetic field is directed along the z-axis, $\vec{H}(\vec{r}) = H(x)\vec{e}_{\mathrm{z}}$. The propagation direction of the wave is along the x-axis. Since the fields depend only on the x-coordinate, the one-dimensional Helmholtz' equations have the form [4]

$$\frac{d^2E}{dx^2} - \frac{1}{\mu}\frac{d\mu}{dx}\frac{dE}{dx} + \omega^2\mu\epsilon E(x) = 0 \quad , \quad \frac{d^2H}{dx^2} - \frac{1}{\epsilon}\frac{d\epsilon}{dx}\frac{dH}{dx} + \omega^2\mu\epsilon H(x) = 0 \quad , \tag{1}$$

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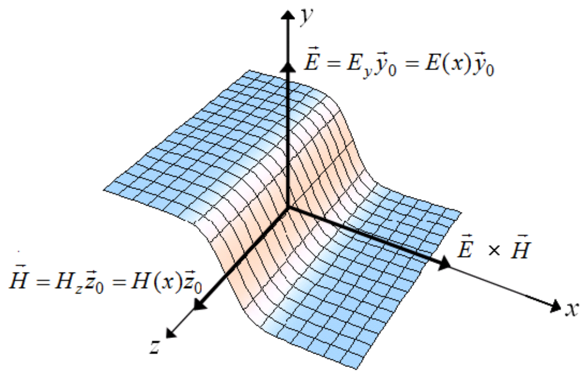

Fig. 1: Propagation of a wave through a graded index structure with a hyperbolic tangent profile.

where $\epsilon = \epsilon(\omega, x)$ and $\mu = \mu(\omega, x)$ are the frequency-dependent and stratified dielectric permittivity and magnetic permeability, respectively.

3. Analytical solutions of the field equations

We assume an inhomogeneous medium for which the effective permittivity and permeability vary according to following hyperbolic tangent functions

$$\mu(\omega, x) = -\mu_0 \mu_R \tanh(\rho x) - i\mu_0 \left[\frac{\mu_{I1} + \mu_{I2}}{2} - \frac{\mu_{I1} - \mu_{I2}}{2} \tanh(\rho x)\right] \quad , \tag{2}$$

$$\epsilon(\omega, x) = -\epsilon_0 \epsilon_R(\omega) \tanh(\rho x) - i\epsilon_0 \left[\frac{\epsilon_{I1} + \epsilon_{I2}}{2} - \frac{\epsilon_{I1} - \epsilon_{I2}}{2} \tanh(\rho x)\right] \quad , \tag{3}$$

where ρ is a parameter describing the steepness of the transition from the RHM material at $x < 0$ to the LHM material at $x > 0$. For passive materials, we require $\epsilon_{I1}, \epsilon_{I2} > 0$ and $\mu_{I1}, \mu_{I2} > 0$. A constant wave impedance throughout the structure, requires that the real and imaginary parts of the effective permittivity and permeability satisfy the condition

$$\beta(\omega) = \frac{\mu_{I1} + \mu_{I2}}{2\mu_R - i(\mu_{I1} - \mu_{I2})} = \frac{\epsilon_{I1} + \epsilon_{I2}}{2\epsilon_R - i(\epsilon_{I1} - \epsilon_{I2})} \quad . \tag{4}$$

When the condition (4) is satisfied, we have

$$\mu(\omega, x) = -\mu_0 \frac{\mu_{I1} + \mu_{I2}}{2\beta}(\tanh(\rho x) + i\beta) \quad , \quad \epsilon(\omega, x) = -\epsilon_0 \frac{\epsilon_{I1} + \epsilon_{I2}}{2\beta}(\tanh(\rho x) + i\beta) \quad , \tag{5}$$

Note that the wave impedance $Z = Z_0 Z(\omega) = \sqrt{\mu(\omega, x)/\epsilon(\omega, x)}$ is constant throughout the entire structure and there is no reflection on the graded interface between the two materials. The two differential equations (1) have the exact solutions

$$E(x) = E_0 \, e^{-\kappa\beta x} \left[2\cosh(\rho x)\right]^{i\frac{\kappa}{\rho}} \quad , \quad H(x) = H_0 \, e^{-\kappa\beta x} \left[2\cosh(\rho x)\right]^{i\frac{\kappa}{\rho}} \quad , \tag{6}$$

where E_0 and H_0 are the amplitudes of the electric and magnetic fields at the boundary $x = 0$, and

$$\kappa = k + i\alpha = \frac{\omega}{c}\sqrt{\mu_R \epsilon_R} + i\frac{\omega}{2c}\sqrt{\frac{\epsilon_R}{\mu_R}}(\mu_{I2} - \mu_{I1}) \quad , \tag{7}$$

We note that in the absence of losses, the results (6) are reduced to the results in reference [4] as a special case. The field amplitudes are related by $E_0 = Z_0 Z(\omega) H_0$. The exact solutions (6) are valid for

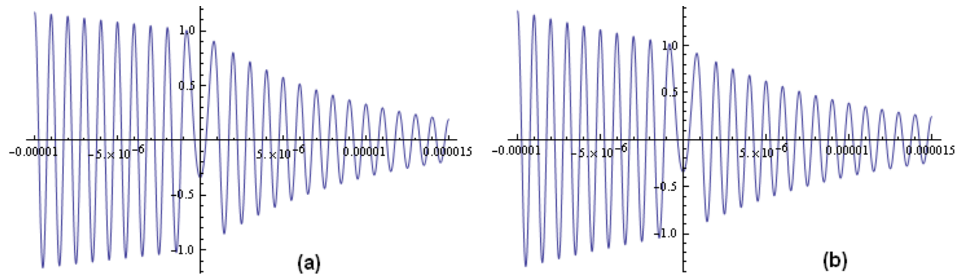

Fig. 2: Analytical results for electric field $E(x)$ as a function of x, with $E_0 = 1$, $k = 2\pi/(10^{-6}m)$, $\rho = 1/(10^{-6}m)$, $\kappa\beta = 2\pi/(10^{-4}m)$ as well as (a) $\alpha = 0.75\kappa\beta$ and (b) $\alpha = 0.50\kappa\beta$.

arbitrary steepness ρ of the graded index interface and arbitrary losses. In the RHM material, we obtain for $x \to -\infty$

$$E(x,t) \sim E_0\, e^{-\gamma_1 x} \cos(\omega t - kx)\ ,\ H(x,t) \sim H_0\, e^{-\gamma_1 x} \cos(\omega t - kx)\ ,\ \gamma_1 = \frac{\omega}{c}\sqrt{\frac{\epsilon_R}{\mu_R}}\mu_{I1} \quad (8)$$

In the LHM material, we obtain for $x \to +\infty$ that

$$E(x,t) \sim E_0\, e^{-\gamma_2 x} \cos[\omega t - (-k)x]\ ,\ H(x,t) \sim H\, e^{-\gamma_2 x} \cos[\omega t - (-k)x]\ ,\ \gamma_2 = \frac{\omega}{c}\sqrt{\frac{\epsilon_R}{\mu_R}}\mu_{I2} \quad (9)$$

For $x \to -\infty$, it follows (8) that the wave in the RHM with the wavevector $\vec{k}_{RHM} = +k\vec{e}_{\mathrm{x}}$ propagates in the $+x$-direction. For $x \to +\infty$, it follows (9) that the wave in the LHM with wavevector $\vec{k}_{LHM} = -k\vec{e}_{\mathrm{x}}$ propagates in the $-x$-direction. The energy flux (the Poynting vector) is still in the $+x$-direction in both media as expected.

4. Graphical presentation and discussion of the results

The exact analytical solutions for the electric field $E(x)$, given by Eq. (6), for two different values of the numerical parameters are presented in Fig. 2. From Fig. 2, we see that there is no reflection at the interface between RHM and LHM, as expected, since the impedance is constant throughout the entire space. However we see that the loss factors γ_1 and γ_2 in RHM and LHM respectively are different.

5. Conclusion

An exact analytical solution to lossy Helmholtz' equations with graded profile, changing according to a hyperbolic tangent function along the direction of propagation, is presented. The expressions and graphical results for the field intensities along the graded structure have been presented. The model allows for arbitrary temporal dispersion and arbitrary loss factors.

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