Metaboundary materialization with extreme anisotropy

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Abstract

This paper focuses on synthesizing metasurfaces and complex boundary conditions. In particular, it is shown how extremely anisotropic materials can be exploited to solve this problem. An example demonstrates how a DB surface can be materialized from an anisotropically covered conductor plane.

1 Introduction – boundary conditions

The relation between electromagnetic boundary conditions and material media is in the very core of metamaterials research. The coupling becomes especially relevant in connection with so-called extreme-parameter materials [1].

In the process of solving electromagnetics and other mathematical physics problems, boundary conditions to characterize given surfaces are important tools in the analysis of fields in a given region. The use of boundary conditions can therefore be termed as an *analytic* approach. The reverse aspect, the *synthetic* approach takes the boundary condition as given, and asks what kind of material structure would mimic the electromagnetic behavior of this boundary, for all kind of excitations. Here, it is essential to distinguish the different levels of materialization in this synthetization procedure: first a given (meta)material setting with given effective constitutive relations to replace the surface, then the metamaterial to be designed by a collection of interacting dipole moments and scattering centers, and finally to realize these elements by artificial molecules to be fabricated with real-world materials.

The impedance boundary condition fixes the ratio of the tangential electric and magnetic field amplitudes by a constant impedance. Well-known examples are the perfect electric (PEC) and perfect magnetic (PMC) conductors, for which this impedance is zero or infinity, respectively. The impedance can also be a two-dimensional dyadic, leading to anisotropic impedance boundaries, and in the extreme case a perfect anisotropy leads to the so-called SHS-boundary (soft-and-hard-surface) [2]. Another extreme possibility is that the surface impedance dyadic is antisymmetric, in which case we are faced with the so-called PEMC boundary [3]. On the other hand, conditions restricting the *normal* components of the fields (or rather, fluxes instead of the fields themselves) on the boundary have been of interest in the recent literature: the DB, D'B', and their relatives, the mixed-impedance surfaces [4] form a fundamental class of boundary conditions. Finally, a recent innovation is the so-called SHDB boundary condition [5] which bridges the tangential and normal boundary conditions in a manner where the DB condition and the SHS condition appear as special cases in a setting of metasurfaces.

The simplest attempt to encounter the above-sketched synthetization question is to try to replace the surface by a semi-infinite volume of homogeneous material with given electromagnetic parameters. A prime example is the task to realize the PEC boundary condition. The obvious procedure is to take a sample of PEC material on the surface of which the PEC boundary condition exists. However, here again a PEC medium is an idealization, and it is by no means trivial to see what type of realistic material

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parameter values would approximate its response [6]. Parameters have to be extreme, but there are several degrees of freedom: isotropy vs. anisotropy, lossless vs. lossy, non-magnetic vs. magnetically active, etc. However, when the synthesis is not limited to homogeneous volumetric materializations, the increased number of degrees of freedom give a lot of possibilities to satisfy novel boundary conditions.

2 Wave-guiding medium

A PEC surface, covered by a layer of medium with lossless material parameters (permittivity ε and permeability μ), acts as a PMC surface, located on the top of the insulating layer, if the thickness of the layer is a quarter-wavelength. However, this effect is only valid for plane waves that meet the surface with normal incidence. For other incidence angles (and hence for any source with a spatial wave spectrum), the impedance is no longer PMC, it rather depends on the wave vector and the polarization of the wave.

This shortcoming can be relaxed by using a slab of so-called wave-guiding medium instead of a plain isotropic layer. A wave-guiding medium is a strongly uniaxial anisotropic medium where the axial components of the permittivity and permeability dyadics ($\varepsilon_{\rm z}$ and $\mu_{\rm z}$) are extremely large. The idea behind such a medium is to force the propagating waves to degenerate into TEM fields, in order to suppress the dependence on the incidence angle for any polarization.

When a surface is covered by a layer of such wave-guiding medium with thickness d in z-direction, the effect is that the surface impedance is transformed to another one [7]. Then, for example to synthesize a perfect PMC surface when the bottom is a conducting metal layer, the condition for the quarter-wave thickness of the layer is

$$\omega\sqrt{\mu_{\rm t}\varepsilon_{\rm t}} = \pi/2 \tag{1}$$

where the angular frequency is ω and the permittivity and permeability are their transversal components. Note here that the layer can be made very thin by increasing the product $\mu_{\rm t}\varepsilon_{\rm t}$. Such a matching sheet could be mounted conformally on curved PEC objects. And most importantly, the capacity of impedance transformation is increased greatly when the transverse permittivity and permeability are allowed both to be 2×2 dyadics. This gives eight degrees of freedom with which a surface impedance with four parameters are to be synthesized.

Examples of metasurface realizations using this principle are the transformations of PEMC from PEC [8], and D'B' from DB [9].

3 Other anisotropies

The strongly uniaxial medium used as transformer in the previous section can be contrasted with one of the materializations of the DB boundary, the ZAP medium [10]. ZAP is another uniaxial extreme-parameter material in which the axial permittivity and permittivity vanish (Zero-Axial-Parameters). On the surface of a transversal cut (perpendicular to the optical axis) of ZAP medium, the normal components of the electric and magnetic flux densities vanish ($\mathbf{n}\cdot\mathbf{D} = 0, \mathbf{n}\cdot\mathbf{B} = 0$) due to continuity of the displacement vectors.

A hybridization of the wave-guiding and ZAP principles opens up a wide range of new possibilities to effectively synthesize complicated boundary conditions. As an example of these possibilities, consider the a conducting (PEC) plane on which lies a quarter-layer of uniaxial medium with optical axis normal to the plane. Assume that the axial parameters are extreme in the following way: $\mu_{\rm z} \to 0$ and $1/\varepsilon_{\rm z} \to 0$. Then it happens that the surface acts like a DB boundary! A numerical test for the reflection from such a layered surface is quantified in Figure 1. There it can be seen that the reflection characteristics approach those of a DB surface, when the axial parameters $\mu_{\rm z}$ and $1/\varepsilon_{\rm z}$ decrease in amplitude (the TE reflection (perpendiculr polarization) resembling that of PEC reflection coefficient ($R = -1, \arg\{R\} = \pi$), and the TM reflection (parallel polarization) approaching PMC reflection ($R = +1, \arg\{R\} = 0$), polarization). Hence this structure operates effectively as a DB surface.

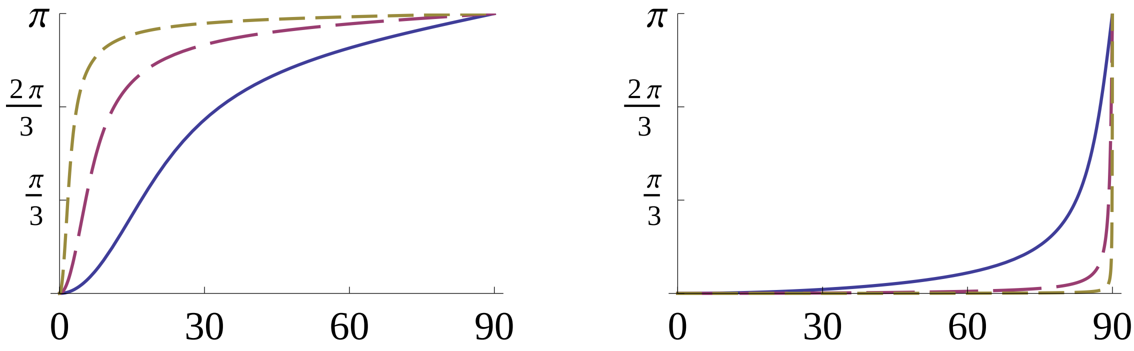

Fig. 1: The argument of the reflection coefficient for the structure of an uniaxially anisotropic layer on a PEC surface as function of the incidence angle in degrees. Left panel: the incident field is TE-polarized (perpendicular, horizontal polarization). Right panel: the incident field is TM-polarized (parallel, vertical polarization). The axial relative parameters are: solid: $\varepsilon_z = 1/\mu_z = 10$, long-dashed: $\varepsilon_z = 1/\mu_z = 100$, short-dashed: $\varepsilon_z = 1/\mu_z = 1000$. Note that the reflection approaches that of a DB surface: PEC-behavior for TE-polarization, and PMC-behavior for TM.

4 Conclusion

A metasurface can be thought as a condensate of the response of a volumetric material in the emerging spirit of metamaterials [11]. Then also, like always in emergence processes where a phenomenon arises from a complex structure with given base properties, the synthesis problem is not unique. Given metasurfaces can be multiply realized. However, since the synthetic approach is more difficult (due to its inverse nature) than the analytic one (where a given structure can be solved in a forward fashion), there is no straightforward recipe to find a realization for a given metasurface. The discussion in this article showed the variety of ways how the problem can be approached using extremely anisotropic materials.

References

[1] A. Sihvola, H. Wallén, and I. V. Lindell, "Extreme-parameter materials and boundary conditions," in *Proceedings of Metamaterials,2009*, London, UK, September 2009, pp. 172–174.
[2] P.-S. Kildal, "Definition of artificially soft and hard surfaces for electromagnetic waves," *Electronics Letters*, vol. 24, no. 3, pp. 168–170, February 1988.
[3] I. V. Lindell and A. H. Sihvola, "Perfect electromagnetic conductor," *Journal of Electromagnetic Waves and Applications*, vol. 19, no. 7, pp. 861–869, 2005.
[4] H. Wallén, I. V. Lindell, and A. Sihvola, "Mixed-impedance boundary conditions," *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 5, pp. 1580–1586, May 2011.
[5] I. V. Lindell and A. Sihvola, "Skewon–Axion medium and Soft-and-Hard/DB boundary conditions," *ArXiV:1201.4738v1*, 2012.
[6] A. Sihvola, I. V. Lindell, H. Wallén, and P. Ylä-Oijala, "Material realizations of Perfect Electric Conductor objects," *Applied Computational Electromagnetics Soc. J.*, vol. 25, no. 12, pp. 1007–1016, December 2010.
[7] I. V. Lindell and A. H. Sihvola, "Realization of the impedance boundary," *IEEE Transactions on Antennas and Propagation*, vol. 54, no. 12, pp. 3669–3676, December 2006.
[8] H. Wallén and A. Sihvola, "How well can a PEC-backed gyrotropic layer approximate the ideal PEMC boundary?" in *Proceedings of EUCAP 2006, 1st European Conference on Antennas and Propagation*, H. Lacoste and L. Ouwehand, Eds., ESA Publications Division, November 2006, paper 349675hw (6 pages).
[9] I. V. Lindell, J. Markkanen, A. Sihvola, and P. Ylä-Oijala, "Realization fo spherical D'B' boundary by a layer of wave-guiding medium," *Metamaterials*, vol. 5, no. 4, pp. 149–154, 2011.
[10] I. V. Lindell and A. H. Sihvola, "Zero-axial-parameter (ZAP) medium sheet," *Progress in Electromagnetics Research*, vol. PIER 89, pp. 213–224, 2009.
[11] A. Sihvola, "Metamaterials in electromagnetics," *Metamaterials*, vol. 1, no. 1, pp. 2–11, 2007.