Negative Refraction by a Hyperbolic Metamaterial formed by Helical-Shaped Wires

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Abstract

We demonstrate that a racemic array of helical-shaped metallic wires may behave as a local uniaxial Epsilon-Negative (ENG) material. It is shown that by increasing the geometrical inductance of the wires it is possible to nearly suppress the spatial dispersion effects that are inherent to wire media at low frequencies, which in turn favors the emergence of negative refraction.

1. Introduction

The phenomenon of negative refraction continues to attract a great deal of attention due to the fact that it contradicts the long-established empirical laws of refraction. This effect may be observed at the interfaces between common dielectrics and materials with simultaneously negative permittivity and permeability [1-2]. There are, however, alternative routes to achieve negative refraction. For example, one possibility is to use indefinite anisotropic (hyperbolic) media, for which the principal components of the permittivity and/or permeability tensors have different signs [3]. In particular, negative refraction based on an indefinite anisotropic material was some time ago demonstrated in the optical regime using an array of metallic nanorods [4]. However, such a configuration is only effective at optical frequencies where the metals exhibit a plasmonic-type response. At lower frequencies (microwave and low infrared frequencies), owing to the large conductivity of the metals, the array of nanowires has a strongly spatially dispersive response [5]. Consequently, the structure behaves quite differently from a material with indefinite parameters and thus the negative refraction is hindered in such a range of frequencies.

Despite these difficulties, it was shown [6] that the spatial dispersion effects in wire media may be tamed either by increasing the capacitance or the inductance of the wires. The capacitance may be increased by attaching metallic plates to the wires, whereas the inductance may be increased by coating the wires with a magnetic material. Even though very effective, the two approaches have drawbacks: attaching plates to the wires may increase dramatically the transverse permittivity of the medium, while coating the wires with a magnetic material is not practical. Quite different from the approach proposed in Ref. [6], or of the idea of using lumped inductive loads [7] which are available only in a limited frequency range, here we suggest increasing the inductance of the wires by geometrical means, and we prove that this strategy provides an effective way to obtain a medium with a hyperbolic local response.

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2. Homogenization Model

The metamaterial considered here consists of a racemic array of perfectly electrically conducting (PEC) helical-shaped wires oriented along the *z*-direction (Fig. 1). Since the structure has a center of symmetry, the response of the metamaterial is non-bianisotropic.

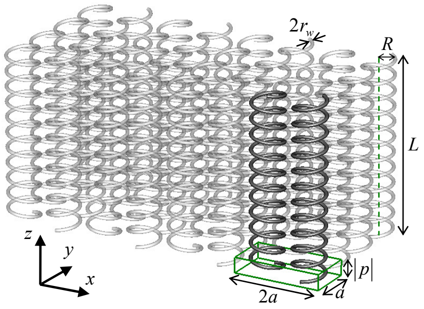

Fig. 1: Geometry of the racemic helical wire medium: a periodic array of PEC helical-shaped wires arranged in an orthorhombic lattice ($2a$ along the *x*-direction, a along the *y*-direction, and $|p|$ along the *z*-direction). Each unit cell includes one right-handed helix and one left-handed helix. R represents the radius of the helices and r_w is the radius of the wires. The helices stand in free-space.

Based on the homogenization approach proposed in Ref. [8], it can be proven that the relative effective permittivity of the considered medium is of the form $\overline{\overline{\varepsilon_r}} = \varepsilon_t\left(\hat{\mathbf{u}}_x\hat{\mathbf{u}}_x + \hat{\mathbf{u}}_y\hat{\mathbf{u}}_y\right) + \varepsilon_{zz}\hat{\mathbf{u}}_z\hat{\mathbf{u}}_z$, with:

$$\varepsilon_t = 1 + \frac{(\pi R)^2}{V_{\text{cell}}C_1},\ \varepsilon_{zz} = 1 - \frac{1}{k_0^2/\beta_{p1}^2 - k_z^2/\beta_{p2}^2} + \frac{A^2k_0^2}{\left(1 + \dfrac{A^2k_0^2}{k_0^2/\beta_{p1}^2 - k_z^2/\beta_{p2}^2}\right)\left(k_0^2/\beta_{p1}^2 - k_z^2/\beta_{p2}^2\right)^2}. \quad (1)$$

where $k_0 = \omega/c$ and $k_z \leftrightarrow j\,d/dz$ is the *z*-component of the wave vector. The definitions of the remaining structural parameters can be found in Ref. [8]. We will focus our analysis on the study of wave propagation in the *xoz* plane ($k_y = 0$), with magnetic field along the *y*-direction (TM-*z* polarized wave). In this case, the dispersion characteristic of the plane waves supported by the racemic helical-shaped wire medium is given by:

$$k_0^2\varepsilon_t - (\varepsilon_t/\varepsilon_{zz})k_x^2 - k_z^2 = 0, \quad (2)$$

where ε_t and ε_{zz} are given by Eq. (1). It can be shown that the characteristic equation (2) reduces to a polynomial equation of second degree in the variables k_0^2 and k_z^2. Hence, the proposed medium supports two independent plane wave modes with magnetic field along the *y*-direction. The emergence of an additional wave is a consequence of the nonlocal response of the metamaterial. Nevertheless, the contribution of the high-frequency mode is unimportant when $\beta_{p2} >> k_z$. As discussed below, this can be ensured by using helical-shaped wires with short pitch p and large radius R.

3. Suppression of Spatial Dispersion and Negative Refraction

In order to demonstrate the drastic reduction of the spatial dispersion effects in the proposed metamaterial (Fig. 1), we have calculated the dispersion diagrams for propagation along the *z*-direction. In Fig. 2a(*i*) we depict the dispersion characteristic of a racemic helical wire medium with radius $R = 0.4a$ and helix pitch $|p| = 0.2a$. To have a reference against which we can compare these results, we depict in Fig. 2a(*ii*) the dispersion characteristic of the standard wire medium formed by straight wires ($|p| = \infty$). The dispersion curves predicted by the homogenization model (solid lines) are compared against full wave hybrid method results [9] (discrete symbols), showing a good agreement.

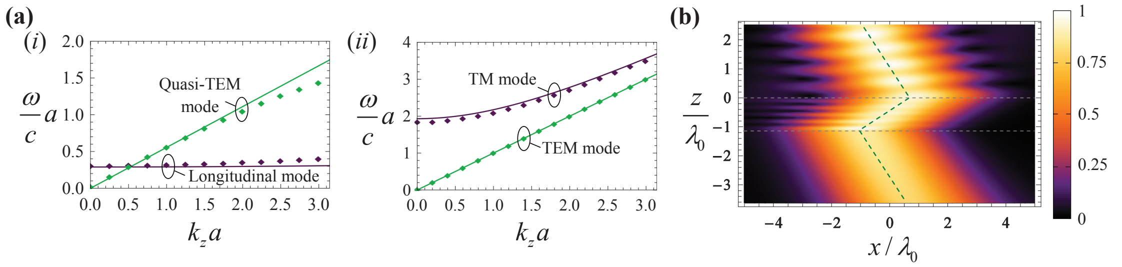

Fig. 2: (a) Dispersion diagrams for the (*i*) racemic helical-shaped wire medium with $R = 0.4a$, $r_w = 0.05a$, and $|p| = 0.2a$, and for the (*ii*) standard wire medium formed by straight metallic wires with $r_w = 0.05a$. Solid lines: nonlocal homogenization results; Discrete symbols: full wave numerical results [9]. (b) Normalized amplitude of the *y*-component of the magnetic field (H_y) inside and outside a racemic helical-shaped wire medium slab with thickness $L = 20a$ illuminated by a Gaussian beam with TM polarization, beam waist $w_0 = 2\lambda_0$, and incident angle $\theta_i = 33^\circ$ at $\omega a / c = 0.35$. The metamaterial has the parameters $R = 0.4a$, $r_w = 0.05a$, and $|p| = 0.3a$.

The suppression of the spatial dispersion effects is evident if we compare the results of Fig. 2a(*i*) and Fig. 2a(*ii*). Indeed, by decreasing the helix pitch p the dispersion of the longitudinal mode is dramatically reduced, so that the slope of the curve becomes almost flat (Fig. 2a(*i*)), and thus the group velocity tends to zero ($v_g = d\omega / dk_z \approx 0$). As a result, in this case the proposed metamaterial behaves as a local hyperbolic material with $\varepsilon_t > 0$ and $\varepsilon_{zz} < 0$. Hence, the metamaterial may negatively refract a *p*-polarized beam of electromagnetic radiation (Fig. 2b). In Fig. 2b we depict the calculated normalized amplitude of the magnetic field inside and outside a helical-shaped wire medium slab, obtained based on the effective medium model. It is seen that the Gaussian beam indeed undergoes a strong negative refraction at both interfaces of the slab. Moreover, it is evident from the plot that the considered metamaterial slab is well-matched to free-space [10]. Hence, the proposed configuration may be an interesting alternative to wire media loaded with metallic patches [11]. At the conference, we will discuss the possibility of achieving a partial focusing with a planar metamaterial lens [10].

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