Power Emitted by a Transverse Dipole Located Above Hyperbolic Medium – Vacuum Interface

C. Guclu[1], S. Campione[1], S. H. Sedighy[1,2], F. Capolino[1]

[1]Department of Electrical Engineering and Computer Science
University of California, Irvine
Irvine 92697, California, USA
Fax: +1-949-824-4152; email: {c.guclu, scampion, sedighis, f.capolino}@uci.edu
[2] School of Electrical Engineering
Iran University of Science and Technology
Narmak, Tehran, Iran
Fax: +98-2177240492; email: sedighy@iust.ac.ir

Abstract

We investigate the power emitted by a transverse dipole at optical frequencies placed at a given distance from the interface between vacuum and a periodic bi-layered hyperbolic metamaterial (HM). We adopt the transmission line formalism for transverse electric (TE) and magnetic (TM) waves to determine the power spectrum emitted by the dipole towards the HM and the fraction of the power directed towards the isotropic half space, at various dipole distances from the interface. For small distances most of the power is directed into the HM, and we investigate the importance of layers' thickness. These results are compared to similar ones in which a transverse dipole is placed above bulk silica-vacuum or bulk silver-vacuum interfaces.

1. Introduction

Hyperbolic metamaterial (HM) is a sub category of uniaxial anisotropic media that exhibits hyperbolic wavenumber dispersion relation. This characteristic dispersion occurs when one of the entries of the permittivity tensor $\underline{\boldsymbol{\varepsilon}}_{\mathrm{HM}} = \varepsilon_t(\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) + \varepsilon_z\hat{\mathbf{z}}\hat{\mathbf{z}}$ modeling the uniaxial anisotropic medium is negative (assuming the anisotropy axis to be along the *z* direction). HM's are proven to yield negative refraction and offer promising features for super-lensing applications [1-3]. Recent interest in HM is also focused on the power emission of a dipole positioned in its proximity. It has been shown that the emitted power of the dipole is mostly directed towards the HM [4], and we justify this result through the information regarding the dipole power spectrum. Moreover ultra-dark absorbers making use of hyperbolic metamaterials are proposed in [5]. Foreseen applications of hyperbolic metamaterials with wide bandwidth and wide angle of operations are, among others, near field absorbers, infrared absorbers, enhancers of scatterers' radiation.

2. Method and Results

A typical way of designing HMs at optical frequency is to alternately stack sub-wavelength-thin layers of materials with negative and positive permittivity, as pictorially shown in Fig. 1(a), with d_1 (d_2) the thickness of the layer with permittivity ε_1 (ε_2). We consider a transverse electric dipole at a distance *h* from the interface between vacuum ($\varepsilon_{\mathrm{up}} = 1$) and a bottom half space at 500 THz. We analyze four different scenarios for the bottom half space: (i) bulk silica (SiO_2, $\varepsilon_{\mathrm{SiO_2}} \approx 2.2$); (ii) bulk silver (Ag, $\varepsilon_{\mathrm{Ag}} \approx -16.05 - j0.44$); (iii) HMd, a HM with silica as the topmost layer; (iv) HMm, a HM with silver

ISBN 978-952-67611-2-1

as the topmost layer. Provided that the HM layer thicknesses are sub wavelength, the homogenization approximation holds as $\varepsilon_t = (\varepsilon_1 d_1 + \varepsilon_2 d_2)/(d_1 + d_2)$ and $\varepsilon_z^{-1} = (\varepsilon_1^{-1} d_1 + \varepsilon_2^{-1} d_2)/(d_1 + d_2)$. Two illustrative cases of HM designs are analyzed: Case 1 has $d_1 = d_2 = 10\text{nm}$; Case 2 has $d_1 = d_2 = 20\text{nm}$; for both cases, $\varepsilon_z \approx 5.10 - j0.02$, and $\varepsilon_t \approx -6.92 - j0.22$. In general, two waves are allowed to propagate in the HM: (i) extraordinary TM waves, whose dispersion relation is given by $k_t^2/\varepsilon_z + k_z^2/\varepsilon_t = \omega^2/c^2$, where k_t and k_z are the transverse to *z* and along *z* wavenumber components, respectively; (ii) ordinary TE waves, mainly evanescent in the HM because $\mathrm{Re}(\varepsilon_t) < 0$. The dispersion diagram for waves in vacuum and the HM are schematically drawn in Fig.1(b).

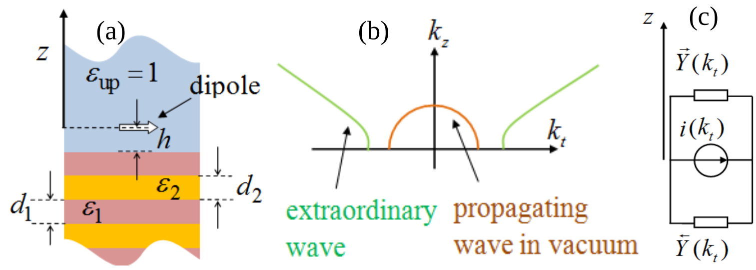

Fig. 1: (a) Bi-layered HM with a transverse electric dipole located in its proximity at a distance *h*. (b) The sketch of the dispersion diagram for waves in vacuum and HM. (c) Transmission line representation.

We utilize the transmission line formalism for both spectral TM and TE waves as shown in Fig.1(c). The transverse dipole is represented as a current generator and the HM is modeled through Bloch theory. The amount of power (*P*) transferred to vacuum (up) or to bottom space (down) is computed through an integration of the spectral power density (*p*) as $P_{\text{up,down}} = |J_x|^2/(8\pi)\int_0^{+\infty} p_{\text{up,down}}(k_t)dk_t$, where *p* is given by (a single (double) prime stands for TM (TE))

$$p_{\text{up}}(k_t) = \mathrm{Re}\left(\frac{\vec{Y}'(k_t)}{\left|\vec{Y}'(k_t)\right|^2} + \frac{\vec{Y}''(k_t)}{\left|\vec{Y}''(k_t)\right|^2}\right)k_t, \qquad p_{\text{down}}(k_t) = \mathrm{Re}\left(\frac{\vec{Y}'(k_t)}{\left|\vec{Y}'(k_t)\right|^2} + \frac{\vec{Y}''(k_t)}{\left|\vec{Y}''(k_t)\right|^2}\right)k_t \quad (1)$$

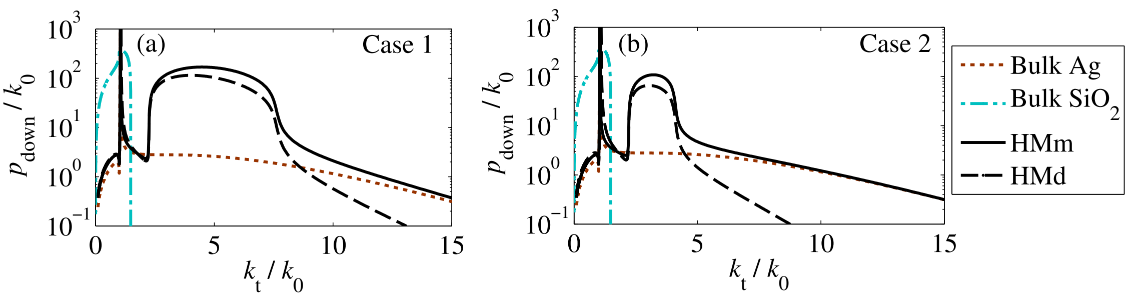

Fig. 2: Normalized spectral power directed downward as a comparison of bulk silica, bulk silver, HMd, HMm, as the lower half space at 500THz for $h = 20$ nm (a) for Case 1, (b) for Case 2.

In Fig. 2, we show the spectral power density directed towards SiO_2, Ag, HMm and HMd for the two cases when the dipole is located at a distance *h* = 20 nm from the interface. The power directed to the vacuum is limited to the propagating spectrum with $k_t/k_0 < 1$ and not plotted here. The spectral power down-directed towards bulk silica is limited to the propagating spectrum with $k_t/k_0 < \sqrt{\varepsilon_{SiO_2}}$. On the other hand, the bulk silver is almost opaque for all the spectral components, because it exhibits negative permittivity. In HM cases, the power is emitted downward for a spectrum band between $k_t/k_0 = \sqrt{\varepsilon_z} \approx 2.3$ and $k_t/k_0 \approx 7.7$ for Case 1, and between $k_t/k_0 = \sqrt{\varepsilon_z} \approx 2.3$ and $k_t/k_0 \approx 4.1$ for Case 2. When $k_t > k_0\sqrt{\varepsilon_z}$, all k_t values correspond to a mainly real k_z, till k_z is large enough that the periodicity of the bi-layered HM is no longer sub wavelength. The periodicity in *z*-direction for Case 2 is twice the period of Case 1, therefore for Case 2 HM-propagation cut-off occurs at lower values of k_t , as shown in Fig. 2.

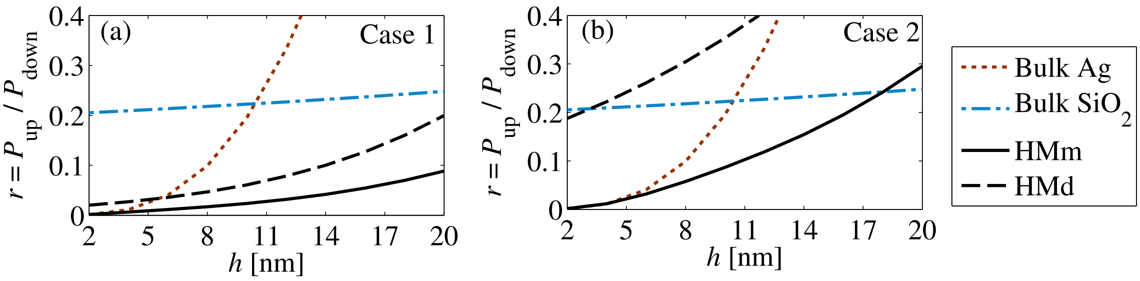

Fig. 3: Ratio of the power directed to vacuum to the total power emitted by the dipole, for SiO_2, Ag, HMm and HMd at 500THz (a) for Case 1, (b) for Case 2.

The ratio ($r = P_{up} / P_{down}$) of the total power directed towards vacuum P_{up} to the total power directed to the bottom space P_{down} is plotted versus the dipole height *h* in Fig.3. When the dipole is close to bulk silver, P_{up} drops significantly due to the destructive interference with the image current, and P_{down} is dissipated as loss. When the dipole is close to SiO_2, since the propagating spectra in vacuum and silica are similar, the ratio *r* changes slowly with respect to *h*. Very small *r* is achieved when the dipole is close to the HM, meaning that most of the power is directed downward, and there absorbed because of Ag losses. Case 1 performs better than Case 2 because of having a wider spectrum that carries power, compare Fig.2 (a, b). Regarding the HMd case, field decay in the topmost dielectric layer without causing losses, and thus *r* is higher than for the HMm case, especially in Case 2. In both HM cases, when *h* increases, *r* also increases, since the spectrum that carries power inside the HM corresponds to the evanescent spectrum of the vacuum. The increase in *h* prevents high k_t components from being coupled to the HM due to evanescent decay in vacuum. This result shows the importance of close proximity of the emitter to the bottom space; more specifically, the use of the HMm is preferred, in order to make use of the wide spectrum of propagation in HM.

3. Conclusion

Hyperbolic metamaterial acts as an *electromagnetic well* when an emitter is placed in its proximity, due to the enhanced scattered field into the HM. Provided that the scatterer is close enough to the interface, the fraction of power directed into the HM is bigger than in the case of bulk dielectric or metal bottom spaces. Importantly, the choice of the topmost layer also affects the fraction of power directed to the HM. Moreover, spectral-wavenumber analysis with transmission line formalism of *H* and *E* waves transverse to the stacking direction of the bi-layered HM is a powerful tool for investigating the power emission of the dipole and its interaction with the HM itself.

References

[1] K. J. Webb and M. Yang, Subwavelength imaging with a multilayer silver film structure, *Optics Letters*, vol. 31, pp. 2130-2132, 2006.

[2] X. Li and F. Zhuang, Multilayered structures with high subwavelength resolution based on the metal-dielectric composites, *Journal of the Optical Society of America A-Optics Image Science and Vision*, vol. 26, pp. 2521-2525, 2009.

[3] Y. Jin, Improving subwavelength resolution of multilayered structures containing negative-permittivity layers by flatting the transmission curves, *Progress in Electromagnetics Research*, vol. 105, pp. 347-364, 2010.

[4] O. Kidwai, S. V. Zhukovsky, and J. E. Sipe, Dipole radiation near hyperbolic metamaterials: applicability of effective-medium approximation, *Optics Letters*, vol. 36, pp. 2530-2532, 2011.

[5] E. E. Narimanov, H. Li, Y. A. Barnakov, T. U. Tumkur, and M. A. Noginov, Darker than black: radiation-absorbing metamaterial, *arXiv*, 1109.5469v1, 2011.