Challenges of metamaterial homogenization in dispersive cloaking shells

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Abstract

The problem of metamaterial homogenization is complicated by coupling and splitting phenomena in finite arrays of close-packed resonators, which form, in particular, invisibility cloaks. The presented results demonstrate an opportunity to provide coherent responses at superluminal wave propagation in the cloaking media comprised of dielectric resonators.

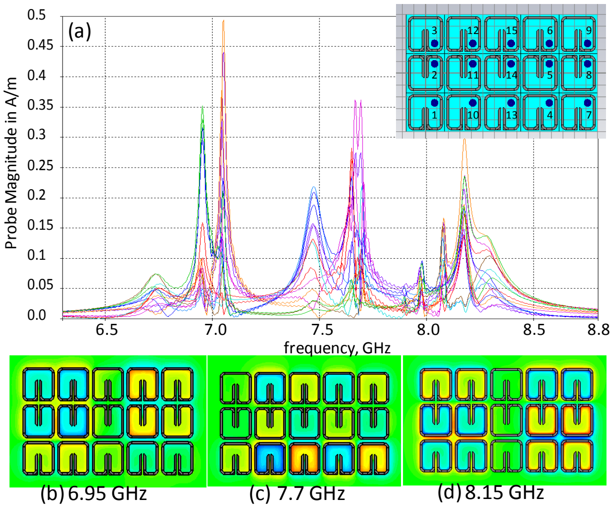
1. Introduction

Metamaterials are usually considered as homogenized media, in which all particles are excited at a single resonance frequency and the formed magnetic/electric dipoles are identically oriented. In such media, no resonance splitting phenomena caused by coupling between resonators could be expected. Simulations of metamaterial responses usually performed for plane wave incidence on a unit cell with periodic boundary conditions in two directions normal to the wave vector *k*, do not reveal resonance splitting. These simulations, however, can only describe an infinite 2D array of unit cells, while stacking 2D arrays in the direction of wave propagation demonstrates quite different results. As it was shown in [1], extracted effective material parameters of coupled fish-nets piled-up in the wave propagation direction had little in common with the data obtained for a single fish-net. Stacked infinite arrays of dielectric resonators also demonstrated responses different from the response of a unit cell with a single resonator [2]. Simulations of finite 3D metamaterial arrays have revealed even more drastic coupling phenomena, when wave propagation along the chains of coupled resonators has been observed [3].

The described results call for a deeper investigation of multi-resonance responses from cloak metamaterials comprised of close-packed arrays of resonators, which are infinite only in one direction. This work aims to clarify the challenges of metamaterial homogenization in invisibility cloaks composed as from split-ring resonators (SRRs) [4], so from dielectric resonators (DRs) [5].

2. Coupling phenomena in fragments of SRR arrays employed in the microwave cloak

First, resonance responses provided by fragments of SRRs employed in the microwave cloak design described in [4] have been simulated. The unit cells and the SRRs in the performed simulations had exactly same parameters as in [4]. The only difference was that the fragments investigated here were flat. However, at a proper restriction of the array lengths these fragments provided for a good approximation of their concentric counterparts in [4]. The k-vector of the incident wave in these studies was directed along the SRR rows in the array planes, while magnetic field was normal to these planes. Thus, the investigated arrays had to perform similarly to the cloak fragments in [4] located near the diameter of the shell normal to the k-vector of the incident wave. H-field probes were used to characterize elementary resonances. Simulations were conducted by using the commercial software package CST Microwave Studio.



(b) 6.95 GHz (c) 7.7 GHz (d) 8.15 GHz

Fig. 1. (a) The spectra of the signals of H-probes located in the array as shown in the inset; and (b)-(d) field patterns of the array at 6.64 GHz, 7.4 GHz, and 8.15 GHz (red and blue colours mark opposite phases, colour intensity shows the magnitude of oscillations).

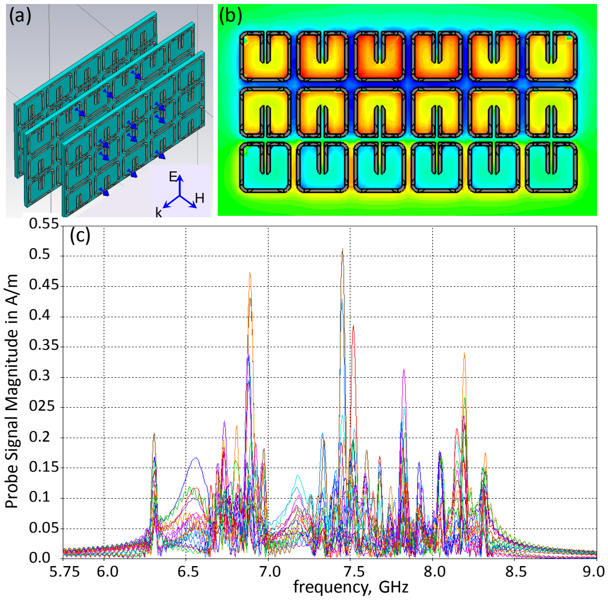


Fig. 2. (a) Geometry of a three-layer array of SRRs with probe locations; (b) coherent response of SRR rows at 8.35 GHz; and (c) spectra of probe signals showing enhanced splitting due to SRR interaction.

The basic element of SRR arrays in [4] was a column of three SRRs with face-to-face and back-to-back interfaces. It appeared that the response of a single column is split in three resonances covering the range of 1 GHz. The 1st resonance was accompanied by a coherent response of the upper pair of SRRs, the 2nd one - by a coherent response of the pair formed by the upper and the lower SRRs, and the 3d one – by a coherent response of the lower pair of SRRs. Combining five columns in one array led to appearance of three groups of split resonances originated from three resonances in columns. Specific splitting of responses within these groups could be related to the intercolumn coupling (Fig. 1). More drastic splitting was observed in the response of a three-layer cloak fragment, formed by planar arrays, each comprising five or six columns (Fig. 2). Resonance peaks in this case were spread over an essentially wider band exceeding 2 GHz. Such stretching could be unequivocally interpreted as a result of strong coupling between parallel planar arrays of SRRs. Similar coupling effects should be expected between concentric arrays of SRRs in the cloak design. This could exclude a possibility for the concentric layers to provide independent responses and could affect the homogenisation within each of the layers. The obtained results also cause doubts in the possibility to represent responses of concentric layers of the cloak by responses of unit cells of infinite planar 2D array as it was considered in [4]. The observed splitting could also disturb radial dispersion of the effective medium permeability prescribed by the transformation optics relations.

3. Homogenization in DR arrays chosen for application in the microwave cloak

The simulation model of the dielectric structure was represented by finite planar 2D array with periodic boundary conditions applied in the normal to the array plane direction (Fig. 3a). The k-vector of the incident plane wave was perpendicular to the infinite Y-axis and oriented along Z-axis. For simplicity, a cubic structure of the arrays was employed. Individual responses of resonators in mutually perpendicular linear arrays of resonators were controlled by the probes placed in the centres of respective resonators. In addition, more probes were placed at some distances from the array to detect transmission. Wave propagation through arrays was controlled by using snap-shots and animation of H-field distribution patterns in XZ cross-section of the array.

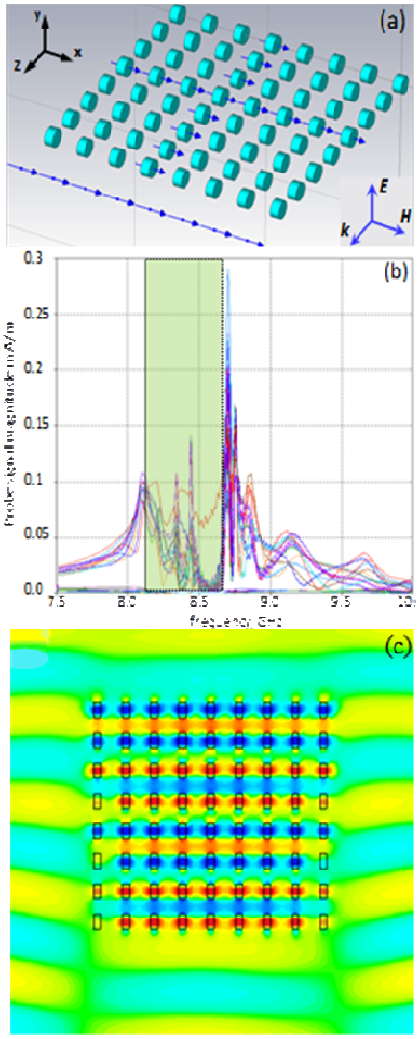


Fig. 3. (a) Planar 2D DR array with probe locations; (b) spectra of probe signals in DRs revealing transmission gap; and (c) snap-shot of wave propagation through array at 8.7 GHz.

As seen in Figs. 3b, the response of array elements is characterized by multiple resonances observed, in particular, in part of the transmission gap marked by green colour. Strong coupling and splitting phenomena are obvious from the presented results. However, these phenomena seem not to affect significantly the response of the resonator array at frequencies below and above the transmission gap, when waves pass through the array. At most frequencies in these ranges coherent responses of resonators are observed within half wavelengths of passing waves (Fig. 3c). It is worth noting that below the gap these responses are usually in phase with the incident wave, while above the gap they are in opposite phase with the wave. In addition, below the gap, the lengths of waves in the array are smaller than that in free space, and the difference is the more, the closer to the gap is the sampled frequency. Above the gap, *vice versa*, the lengths of waves exceed the wavelengths in free space (Fig. 3c), so that the waves travelling through the array outride those moving in free space. This means that the waves move with superluminal phase velocity. The measurements of the wavelengths can be used to estimate the effective parameters of the medium and these estimates showed that below the gap, a strong increase of the effective permeability occurs, while above the gap, the effective permeability values are in the range from 0 to 1.The described specifics justify employment of DR arrays in cloaking shells for obtaining invisibility effect at frequencies above the gap without serious concerns about distorted homogenisation due to the resonator coupling.

4. Conclusion

Coupling between resonators in cloak metamaterials can complicate obtaining homogenized media with prescribed spatial dispersion at close packing of resonators in the SRR based cloak. In DR arrays, coupling and splitting phenomena appear localized mainly within the transmission gap that facilitates achieving homogenization at frequencies, when the array demonstrates singular effective parameters required for cloaking.

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