# Mesoscopic surface effects in finite metamaterial samples

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**Abstract**

We report on mesoscopic effects associated with the boundaries of finite discrete metamaterial samples, which can invalidate an effective medium description. We make a proposal in order to avoid such effects by a proper choice of the boundary configuration.

Usually, effective medium homogenization theories of metamaterial structures consider unbounded systems. However, all metamaterial bodies must have finite size. While it can be expected that extremely big samples of metamaterials (with perhaps millions of elements) will show a behavior quite similar to that of the unbounded medium, this is not clear for realistic metamaterial samples (up to thousands of elements). This raises the issue of the surface resonances, which may appear even in the quasi-static regime and can significantly deviate from the behavior expected for the effective medium [1], [2].

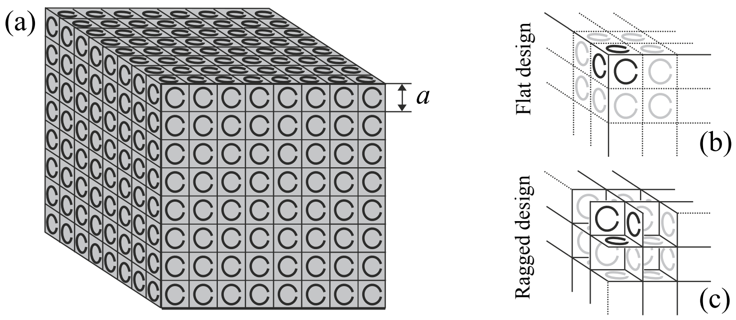


Fig. 1: (a) Sketch of the cube with the corresponding coordinate system. Note that the actual number of elements may vary which is not reflected in this sketch. (b, c) Two options for the isotropic surface configuration (detail near the corners): "flat" geometry (b) and "ragged" geometry (c)

In order to elucidate this question, we analyzed the behavior of a cubic sample of metamaterial made of a finite cubic lattice of resonant rings. This sample is sketched in Fig.1, and is made of symmetric resonant rings which, in practice, can be made (at low frequencies) by simply loading a metallic ring with a capacitor [3]. In order to analyze surface resonances, we tried with two different terminations, which will be named as "flat" and "ragged", also shown in Fig.1: the "flat" geometry includes a external layer of rings, which is absent on the "ragged" termination. Finally, in order to have an electrically small size for the unit cell, we studied a system with strongly subwavelength rings with the lattice constant $a$ 50 times smaller than the free-space wavelength, and radius of the ring $r_0 = a/3.1$. The quality factor of the resonators has been chosen rather high ($Q = 500$) in order to achieve better clarity in the results.

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A convenient macroscopic characteristic that can be attributed to the studied samples is the normalized polarizability: the total magnetic moment of the sample per unit external magnetic field and per unit volume, which for continuous medium samples of subwavelength size is independent of the size. To find the polarizability of our discrete cubes, we solved a system of circuit equations for coupled resonators [4], assuming a uniform external magnetic field perpendicular to the face of the cube. The total magnetic moment is computed by summing the individual magnetic moments of each ring. The results obtained for samples of different size are shown in Fig.2. As it can be seen, there are significant differences between the normalized polarizabilities of samples, without a clear trend to a common limit, specially for the "flat" configuration.

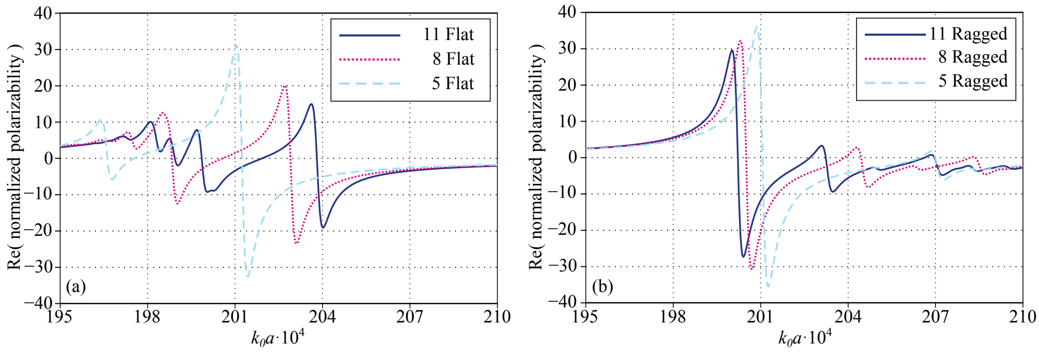


Fig. 2: Real part of the normalized polarizability (arbitrary units) of discrete cubes with 5, 8 and 11 layers of rings in each direction, having either a "flat" (a) or a "ragged" geometry (b).

Further distinctions are revealed by comparing (Fig.3) discrete cubes with an equivalent cube made of a homogeneous medium, with the effective permeability given by Eq. (13) of Ref.[5]. The polarizability of the homogeneous cube was found using the CST "Microwave Studio" commercial package. In order to avoid the appearance of multiple non-physical resonances, the edges and corners of the homogeneous cube must be rounded [6]. We have used a rounding corresponding to one half of the unit cell size of the discrete cubes. Fig.3 shows that, although the polarizability of discrete cubes are different from that of a homogeneous cube, the results for the "ragged" configuration are significantly closer to the results for the homogeneous cube than to those of the "flat" cube.

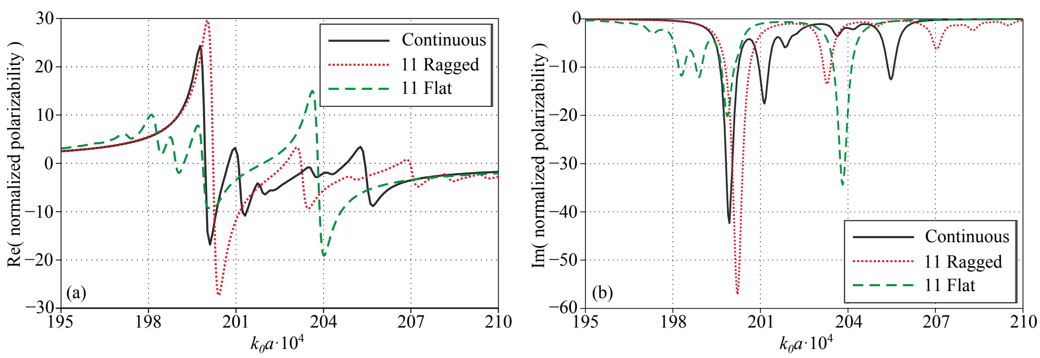


Fig. 3: Real (a) and imaginary (b) parts of the normalized polarizability (arbitrary units) of discrete cubes with 11 layers in each direction, having either a "flat" geometry (dashed lines) or "ragged" geometry (dotted lines), in comparison with the polarizability of a homogeneous cube (solid lines) .

We can now try to retrieve the "permeability" of the samples from their normalized polarizability $\alpha$. For a continuous sample we can write without loss of generality

$$\alpha = A\frac{\mu - 1}{\mu + C} \tag{1}$$

where $A(\omega)$ and $C(\omega)$ are real coefficients which depend on the geometry of the sample (for a spherical sample it is $A = 3$ and $C = 2$, but for a cubic sample they are rather complicated functions of frequency). Therefore, we can try to retrieve the "permeability" of the sample from Eq.1 by using the obtained results for the normalized polarizability, and the coefficients $A$ and $C$ obtained for the continuous sample. These results are shown in Fig.4 for the discrete samples of Fig.3. It is very remarkable that, unlike the curve of the "flat" cube, the curve corresponding to the "ragged" configuration is very similar to the actual permeability of a homogeneous cube.

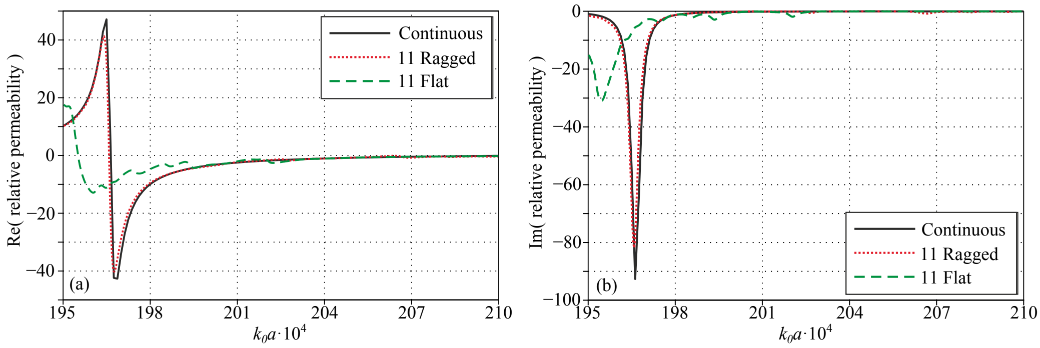


Fig. 4: A comparison between the permeability of a homogeneous cube (solid line) and the "permeability" obtained for two $11 \times 11$ layer discrete cubes with a "flat" geometry (dashed lines) or a "ragged" geometry (dotted lines); real (a) and imaginary (b) parts .

In summary, we have shown that the behavior of finite metamaterial samples can deviate significantly from continuous medium expectations. From our results we conclude that for cubic resonant ring metamaterial samples with up to several thousands of elements, the "ragged" boundary configuration is better described in terms of the effective parameters of an unbounded metamaterial than the "flat" boundary configuration. We are confident that, for the size range in which most practical metamaterials fit, our conclusions provide valuable information and design guidelines, allowing for the efficient control over the properties of finite metamaterial structures. Our results can be also of interest for the general theory of mesoscopic electromagnetic systems.

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## References

[1] R. Marques, M. Lapine, and L. Jelinek, Edge effects in finite size resonant ring metamaterials, in *Proc. Metamaterials Int. Conf. Complex Electromagn. Media Metamater.*, Karlsruhe, Germany, 2010, pp. 330-332.
[2] R. Marques, L. Jelinek, M. J. Freire, J. D. Baena and M. Lapine, Bulk Metamaterials Made of Resonant Rings, *Proceedings of the IEEE*, vol. 99 pp. 1660-1668, 2011.
[3] S. A. Schelkunoff and H. T. Friis *Antennas Theory and Practice* (Wiley), 1966.
[4] M. Lapine, L. Jelinek, R. Marques, M.J. Freire, Exact modelling method for discrete finite metamaterial lens *IET Microw. Antennas Propag.*, vol. 4, pp. 1132-1139, 2010.
[5] J. D. Baena, L. Jelinek, R. Marques and M. Silveirinha, Unified homogenization theory for magnetoinductive and electromagnetic waves in split-ring metamaterials, *Phys. Rev. A*, vol. 78, 013842, 2008.
[6] H. Wallen, H. Kettunen and A. Sihvola, Surface modes of negative-parameter interfaces and the importance of rounding sharp corners, *Metamaterials*, vol. 2, pp. 113-121, 2008

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