Spatial filtering by metasurfaces made of chains of interconnected SRRs and CSRRs

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Abstract

Here we study two metasurfaces made of 1D chains of Interconnected Split Ring Resonators (I-SRRs) or Interconnected Complementary Split Ring Resonators (I-CSRRs). The main result was that the central frequencies of the stopband and passband can be tuned by controlling the incidence angle, while it is constant for SRR/CSRR metasurfaces.

1. Introduction

Since a few years ago, many researchers have been involved in the design of metamaterial Frequency Selective Surfaces (FSS), also called metasurfaces, made of the Split Ring Resonator (SRR) and the Coplementary Split Ring Resonator (CSRR), or similar resonators. Fig. 1(a) and Fig. 1(b) show two examples with the SRR and the CSRR, but note that many other topologies of the unit cell have been used. It is well known that the pass-band filtering feature is obtained for sample with SRRs and stop-band filtering features with CSRRs [1]. It was also demonstrated that these behaviours stay stable under off-normal incidence [2]. The aim of this paper is to investigate the effects of connecting these resonators forming 1D chains as shown in Figs. 1(c) and 1(d). We will demonstrate that the new structures are tuneable, being possible to shift significantly the central frequency while the bandwidth stays constant.

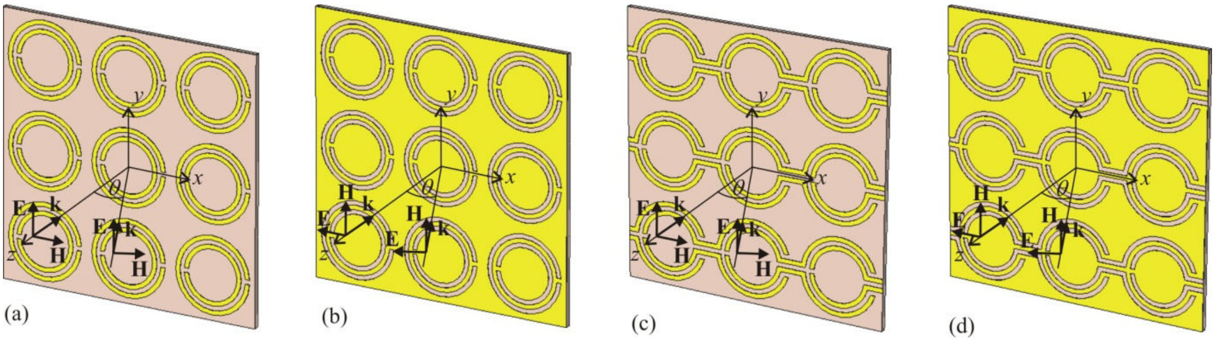


Fig. 1: Metasurfaces made of unconnected SRRs (a), unconnected CSRRs (b), interconnected I-SRRs (c), and unconnected I-CSRRs (d). Yellow represents copper and grey represents the dielectric substrate.

2. Theory

When Pendry and co-workers introduced the SRR in Ref. [3], it was modelled as an RLC circuit. Many other papers have contributed to a more detailed understanding of its behaviour. Here we use the approximations for R_{SRR}, L_{SRR}, and C_{SRR} dictated in Ref. [4]. However the model of the unit cells shown in Fig. 1(c) and Fig. 1(d) must be quite different due to the bifilar lines connecting the chains of SRRs or CSRRs. The new unit cells are going to call as Interconected SRR (I-SRR) and Interconnected CSRR (I-CSRR). Now, please take a look on the diagram of Fig. 2(a) and the circuit model below in Fig. 2(b). The ring drawn with solid line represents one period of the chain, one I-SRR, while the rings in dashed lines are its first neighbors. We are going to consider that the chain is periodical and so infinite. First, note that each ring have been cut just at the point where the electric current is supposed to have a maximum value and, just at the cut, a bifilar line has been soldered in order to trespass current between the first neighboring cells. Thus, if the phase shift between the currents at the left and right bifilar lines was not important, then the resonance of the I-SRR should be quite similar to that of the unconnected SRR. However, this phase shift must be present and the single LC resonance is replaced by a surface mode whose the dispersion relation is

$$\left(\omega/\omega_0\right)^2 = \left(1+L_{\text{line}}/L_{\text{SRR}}\right)^{-1}\left(1+2\left(C_{\text{SRR}}/C_{\text{line}}\right)\left(1-\cos(ka)\right)\right) \tag{1}$$

where ω is the angular frequency, L_{SRR} and C_{SRR} are the inductance and capacitance of the single SRR, ω_0 $\omega_0 = 1/\sqrt{L_{\text{SRR}}C_{\text{SRR}}}$ is the resonance frequency of the SRR, and L_{line} and C_{line} are the inductance and capacitance corresponding to the short piece of bifilar transmission line used in the I-SRR. The demonstration of (1) is straight forward by using Kirchhoff's equations over the periodical circuit cell shown in Fig. 2(b). L_{line} and C_{line} can be approximated by design formulas contained in Ref. [5]. Since this is a first rough approximation we are not taking care of ohmic nor dielectric losses here.

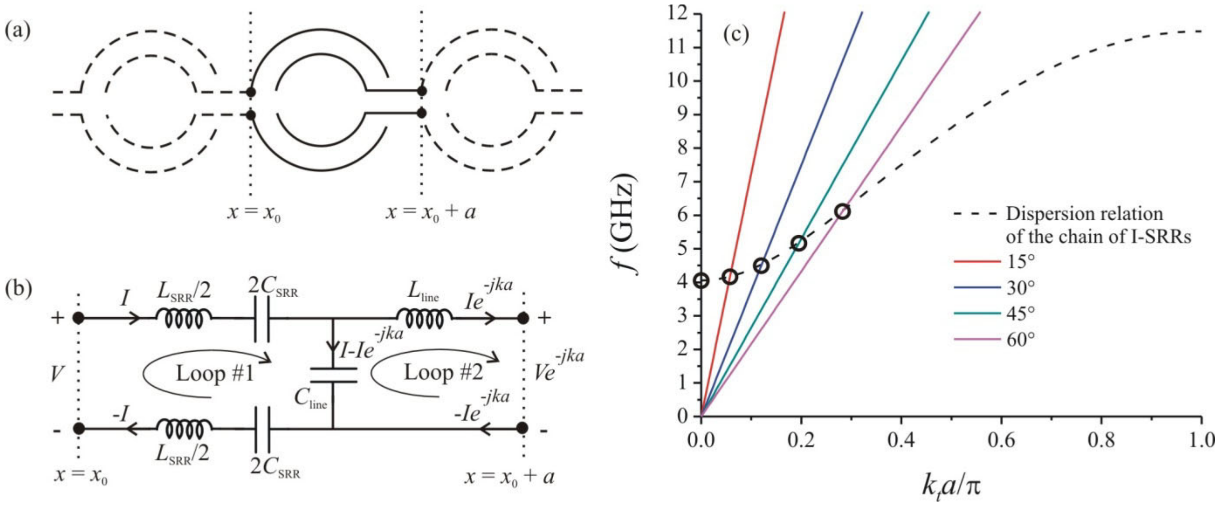
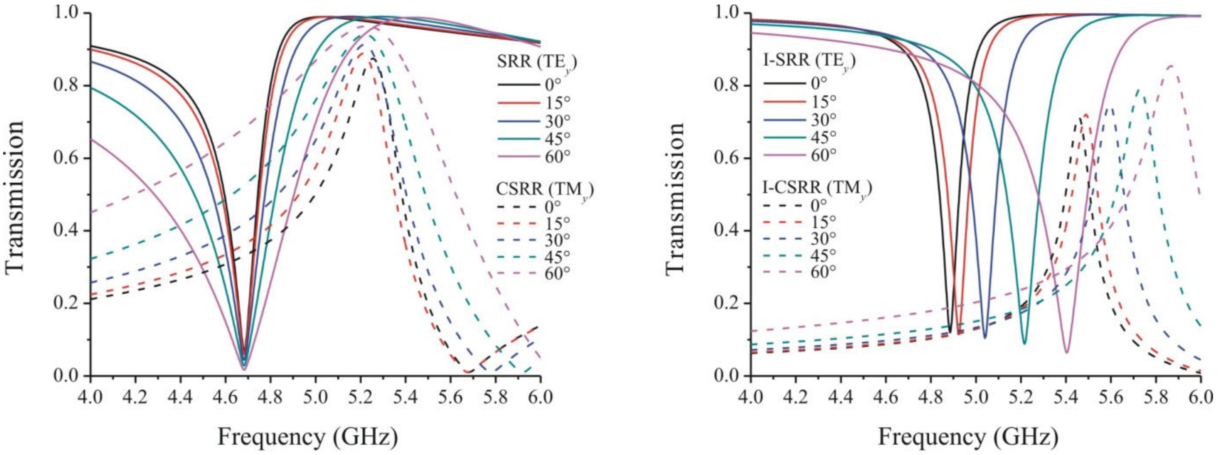


Fig. 2: The Interconnected SRR or I-SRR (a), its circuit model (b), and the corresponding dispersion relation for a 1D periodical chain of I-SRRs (c). Actually, the dispersion relation is the dashed line of (c). Solid lines in (c) represent frequency versus tangential component of the wavevector of the incident wave under different incidence angles. The used parameters were external radius r = 3.5 mm, strip widths w = 0.4 mm, distance between strips d = 0.4 mm, metallic sheet thickness t = 35 μm, dielectric substrate thickness h = 0.49 mm, relative permittivity of the substrate $\varepsilon_r = 2.5(1 - j\,0.003)$, and conductivity of copper.

In Fig. 2(c) the dispersion relation for certain geometrical and electromagnetic parameters is depicted (the dashed line). If now we imagine an incident wave impinging over the surface, it is natural to think that it will strongly couple to the surface mode whose wavevector equals the transverse component of the incident wavevector. Solid lines of Fig. 2(c) are representing frequency versus tangential component of the incident wavevector. Therefore, the dip for the metasurface of I-SRRs in transmission (peak if I-CSRRs are used) should appear just at the frequency where the solid line intersects the dashed line. Since an increment of the incident angle means a decrement of the solid line slope, then a shift to higher frequencies is obtained when the incidence angle is incremented.

3. Numerical results

In order to demonstrate the tuning of metasurfaces of interconnected I-SRRs and I-CSRRs, we have simulated the four geometries shown in Fig. 1 with the physical parameters indicated in the caption of Fig. 2. Simulations were run by using the commercial simulator *CST Microwave Studio*. The results are shown in Fig. 3. It is noticeable that when the rings are unconnected the central frequency does not depend on the incident angle (see Fig. 3(a)), while it does for connected chains of rings (see Fig. 3(b)).



(a) (b)

Fig. 3: Transmission coefficients for metasurfaces made of unconnected SRRs and CSRRs (a) and interconnected I-SRRs and I-CSRRs (b).

4. Conclusion

It has been demonstrated that some designs of metasurfaces made of interconnected resonators have the capability of spatial filtering. We started from old previous designs of filters using the Split Ring Resonator (SRR) and its complementary figure (CSRR) as unit cells. The new idea was to connect them along one direction by using short pieces of bifilar transmission lines. The metasurfaces made of interconnected I-SRRs and I-CSRRs give a stopband and a passband, respectively, whose central frequencies can be tuned by changing the incidence angle without affecting much to the bandwidth.

References

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