# Transformation Media: Space and Time, Curvature and Carpets

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**Abstract**

We present our state of the art unification of Transformation Optics with other transformation-based wave theories (such as acoustics). This formalism allows quite naturally for spacetime devices, as well as for curvature, so we present applications in curved cloaks and spacetime carpets.

## 1. Introduction

We show how it is possible to cast several distinct wave theories (including electromagnetism) into a single prescription [1, 2]. This greatly simplifies the process of learning how to use transformation theories to make transformation devices ("T-devices"), because now you only need to learn one procedure – that for the unified wave theory. Nevertheless, however elegantly they might be expressed, the possible wave theories contained within the formulation are not all equivalent. Thus although the prescription is general, we find that there is a test for whether or not a given T-device is allowed in a particular case. That test is based on the form-invariance of the wave model – if the transformed theory is not the same, perhaps because it demands impossible material properties – then the hoped for T-device is likewise impossible.

Specifically, we will demonstrate how to combine electromagnetism, scalar acoustics, and even some important types of pentamode acoustic waves into a form consisting of a pair of first order differential equations for the behaviour of two rank-2 tensors, linked by a rank-4 constitutive tensor. For the less mathematically inclined, we will also show that a careful rewriting enables us to present the same description much more simply, and entirely in terms of matrices. This is much more general and (arguably) simpler process that has been used in the existing cloak calculations which are specialized to their own particular circumstances (see e.g. [3, 4, 5, 6, 7, 8, 9]).

## 2. Curvature: ray paths and curved surfaces

The ray paths defined by the wave propagation in T-devices are usually changed by the transformation design from a non-curved flat-space reference system into a virtual deformed, *curved* space (or even spacetime). However, it also is perfectly legitimate to start from a curved reference space. Perhaps the simplest example is that of the Maxwell's Fisheye lens [10] – or its "Maxwell's Fishpond" water wave counterpart [11]. This represents wave travel on the (reference) spherical surface by projecting the sphere down onto a flat plane for its implementation. The curvature is then preserved by the resulting non-straight ray trajectories; or in the "Fishpond" case, distortions from circularity in the outgoing ripples, and its restoration as they reconverge to the image point.

Starting from the Maxwell's Fisheye/Fishpond as an exemplar, we will describe how curvature can be induced, preserved, or removed by a transformation procedure. The scheme is outlined on figure 1,

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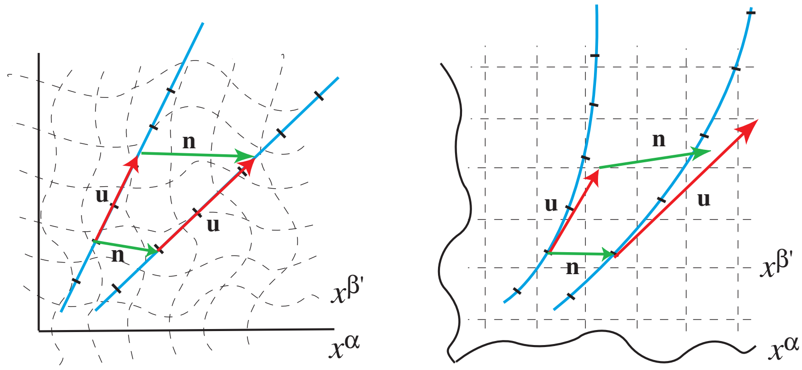


Fig. 1: A clear route between coordinate transformation and induced curvature has been established. After (active) transformation the geodesics of a flat manifold (i.e. straight lines) become curved, and, moreover, accelerate relative to each other.

where, after a transformation is applied, we can insist that the new ray-path curves are geodesics, thus defining a new curved manifold in which the curvature tensor is calculated explicitly in terms of the coordinate transformation matrix $L^{\alpha'}_{\beta} = \frac{\partial x^{\alpha'}}{\partial x^{\beta}}$. We will then show how to apply these concepts in order to generate a cloak design capable of operating on a 2D curved surface.

## 3. Carpets in spacetime

The concept of a spacetime cloak (or "event cloak") [12] is already well established in both theory and experiment. We will show how our unified Transformation Media theory can be applied to novel sorts of carpet cloak. In particular, we will discuss some new spacetime T-devices analogous to the *spatial* ground-plane T-devices that already exist: we show how we can construct electromagnetic spacetime carpet T-devices: We will describe event cloaks, peepholes, (where we cloak a transient dent in the surface), and tardises. Here, a peephole is where we transform a transient dent in the surface so that the surface always looks flat, and a "tardis" is where we temporarily induce the illusion of more spacetime than actually exists. By also attempting to construct acoustic versions, we will elegantly demonstrate the restrictions on T-devices using standard (scalar) acoustic waves, and so explain why more exotic forms of acoustic wave – e.g. pentamode waves [13] – are required for constructing fully 3D acoustic T-devices such as cloaks.

## 4. Summary

Our state of the art unification of Transformation Optics with other transformation-based wave theories (such as acoustics) is described, and applied to a variety of systems, demonstrating the role of curvature and spacetime transformations in T-devices.

# References

[1] Kinsler, P., McCall, M.W., Favaro, A., Transformation Acoustics, Transformation Mechanics: a general theory of transformation devices, poster (Metamaterials '11, Barcelona, 2011).

[2] Kinsler, P., McCall, M.W., Favaro, A., All Kinds of Cloaks, all Kinds of Transformations: a general theory of Transformation Mechanics, talk (AFPAC 2012, Brighton).

[3] Pendry, J.B., Schurig, D. & Smith, D. R., Controlling EM Fields, *Science* **312**, 1780-1782 (2006).

[4] Schurig, D., Mock, J. J., Justice, B. J., Cummer, S. A., Pendry, J. B., Starr, A. F. & Smith, D. R., Metamaterial Electromagnetic Cloak at Microwave Frequencies, *Science* **314**, 977-980 (2006).

[5] Ergin, T., Stenger, N., Brenner, P., Pendry, J.B. & Wegener, M., Three Dimensional Invisibility Cloaking at Optical Wavelengths, *Science* **328**, 337-339 (2010).

[6] Chen, X., Luo, Y., Zhang, J., Jiang, K., Pendry, J.B. & Zhang, S., Macroscopic Invisibility Cloaking of Visible Light, *Nature Communications* **2**, 176, (2011).

[7] Zhang, S., Xia, C. & Fang, N., Broadband Acoustic Cloak for Ultrasound Waves, *Phys. Rev. Lett.*, 024301, (2011).

[8] Torrent, D. & Sanchez-Dehesa, J., Acoustic Metamaterials for New Two-Dimensional Sonic Devices, *New J. Phys.*, **9**, 323, (2007).

[9] Cummer, S. A. & Schurig, D., One Path to Acoustic Cloaking, *New J. Phys.*, **9**, 45, (2007).

[10] Leonhardt, U., Perfect imaging without negative refraction, *New J. Phys.*, **11**, 093040, (2009).

[11] Kinsler, P., Tan, J., Thio, T.C.Y., Trant, C., & Kandapper, N., Maxwell's Fishpond, arXiv:1206.0003, submitted to Eur. J. Phys.

[12] McCall, M.W., Favaro, A., Kinsler, P. & Boardman, A.D., A Spacetime Cloak, or a History Editor, *Journal of Optics*, **13**, 024003, (2011).

[13] Norris, A.N., Acoustic cloaking theory, *Proc. Royal Soc. A*, **464**, 2411 (2008).

 