Optically large three-dimensional directional cloaks: off-normal incidence performance study

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Abstract

In this paper we study a generalization of directional, optically large eikonal-limit cloaks based on conformal maps, to three dimensions. The cloak is a spherical shell filled with a graded isotropic dielectric whose distribution is cylindrically, although not spherically, symmetric. Due to lack of spherical symmetry, the structure has low visibility only for a limited range of incidence angles. We employ a 2.5D full-wave modelling technique, which exploits cylindrical symmetry but still allows arbitrary non-symmetric excitations, to estimate visibility of a cloak with respect to a monochromatic plane wave incident at various angles. With respect to a certain visibility measure, the structure has a reduced visibility for angles up to 3^o.

1. Introduction of the method

Linear wave propagation through inhomogeneous structures of size $R \gg \lambda$ is a computationally challenging problem, in particular when using finite element methods, due to the steep increase of the number of degrees of freedom as a function of R/λ. Fortunately, when the geometry of the problem possesses symmetries, one may choose an appropriate basis in which the stiffness matrix of the discretized problem is block-diagonal, which then enables diagonalization on the block-by-block basis. A particular scenario is the case of a cylindrically-symmetric geometry, where an appropriate basis is the set of cylindrical waves with all possible azimuthal numbers (n). For many wave forms of interest, in particular the free-space plane wave propagating at a relatively small angle w.r.t the symmetry axis, the cylindrical harmonic expansion converges rapidly, and therefore it can be truncated at a relatively small $n=n_{max}$. Each of these cylindrical harmonics propagates through the structure independently of all other harmonics, and therefore the fields associated with that harmonic can be found by solving an essentially two-dimensional PDE problem in the ρ-z (half)-plane.

Below, we obtain analytic formulas for the expansion of a polarized electromagnetic plane wave in terms of cylindrical harmonics, which are the main ingredient to solving the 3D scattering problem. Our frequency sign convention for phasors is $e^{j\omega t}$.

Consider a free-space EM wave incident at angle θ_i with respect to the z-axis, polarized with magnetic field transverse to the z-axis (TM_z polarization), i.e. $H_z = 0$ and

$$\mathbf{E} = E_0\left(\hat{x}\cos\theta_i + \hat{z}\sin\theta_i\right)e^{-jk_0 x\sin\theta_i}e^{jk_0 z\cos\theta_i} \quad . \qquad (1)$$

Making use of the well-known relation,

$$e^{-j\gamma x} = \sum_{n=-\infty}^{\infty} j^{-n} J_n(\gamma\rho) e^{jn\phi}, \tag{2}$$

the *z*-component of **E** can be written as

$$E_z = E_0 \sin\theta_i e^{jk_0 z\cos\theta_i} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k_0\rho\sin\theta_i) e^{jn\phi}. \tag{3}$$

From Ampere’s and Faraday’s laws, we have the following differential relationships for the cylindrical components of fields:

$$H_\rho = -\frac{1}{j\omega\mu}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right), \qquad E_\phi = \frac{1}{j\omega\varepsilon}\frac{\partial H_\rho}{\partial z}. \tag{4}$$

Combining these expressions, one obtains

$$H_\rho = -\frac{1}{j\omega\mu}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{1}{j\omega\varepsilon}\frac{\partial^2 H_\rho}{\partial z^2}\right). \tag{5}$$

Using expansion (3) for E_z, we find the expansion for H_ρ:

$$H_\rho = -j\frac{E_0}{\eta}\frac{e^{jk_0 z\cos\theta_i}}{k_0\rho\sin\theta_i}\sum_{n=-\infty}^{\infty} n j^{-n+1} J_n(k_0\rho\sin\theta_i) e^{jn\phi}. \tag{6}$$

Furthermore,

$$H_\phi = -\frac{1}{j\omega\mu}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial\rho}\right), \qquad E_\rho = -\frac{1}{j\omega\varepsilon}\frac{\partial H_\phi}{\partial z}, \tag{7}$$

and thus,

$$H_\phi = -\frac{1}{j\omega\mu}\left(-\frac{1}{j\omega\varepsilon}\frac{\partial^2 H_\phi}{\partial z^2} - \frac{\partial E_z}{\partial\rho}\right). \tag{8}$$

Finally, we obtain

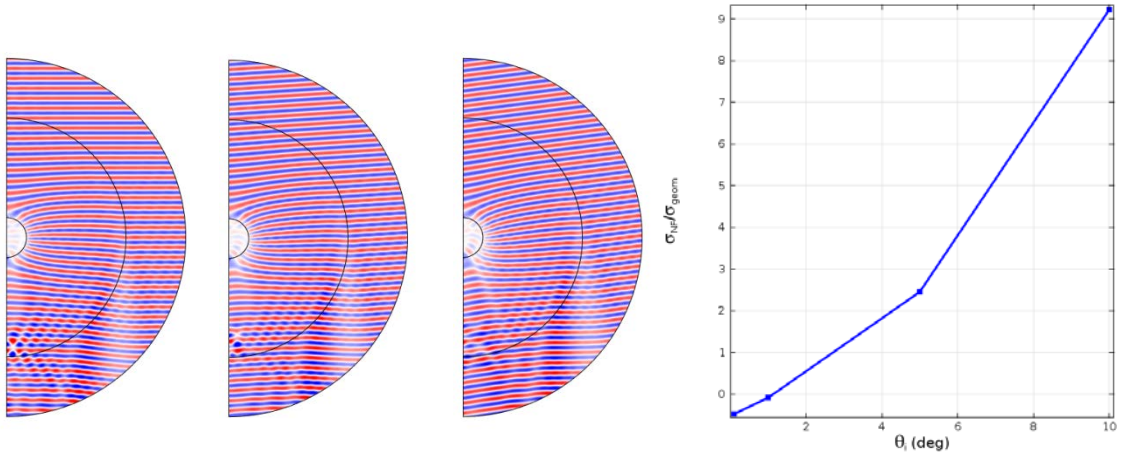
$$H_\phi = j\frac{E_0}{\eta} e^{jk_0 z\cos\theta_i} \sum_{n=-\infty}^{\infty} j^{-n} J_n'(k_0\rho\sin\theta_i) e^{jn\phi}. \tag{9}$$

The expressions (3), (6) and (9) supplemented with (4) and (7) provide cylindrical harmonic expansions of all five non-vanishing field components ($E_z, E_\rho, E_\varphi, H_\rho, H_\varphi$) in the TM_z-polarized wave. The five field components of the TE_z-polarized plane wave are obtained trivially from these expressions using electromagnetic duality, i.e. by swapping E-H and ε-μ.

2. Directional isotropic-medium cloak: performance at various angles

We implement the 2.5D technique in COMSOL Multiphysics (see additional details in Ref. [1]), using the scattered field analysis with customized equations that account for the prescribed φ-dependence of the fields. On the exterior boundary of the simulation domain, we apply the non-reflecting radiation-type boundary condition, modified to allow φ-dependent fields.

The directional isotropic-medium cloak is described in Ref. [1], and it is based on a revolved version of the two-dimensional conformal mapping cloak introduced in Ref. [2] and modelled using full-wave simulations in Ref. [3]. Here, we study the behaviour of the 3D cloak as a function of incidence angle. Since the coordinate map is designed to effectively compress the concealed object (smaller circle in Fig.1a) to a flat sheet, it is expected that its performance should degrade rapidly with increasing incidence angle. For concreteness, we assume the following cloak parameters: cloaked object radius R_1=0.2m, free-space wavelength $\lambda = R_1/2$=0.1m, outer cloak radius R_2=6 R_1=1.2m. The fields obtained from 2.5D models are visualized in Fig.1a-c.



(a) (b) (c) (d)

Fig. 1: Directional 3D cloak: field plots on the φ=45° half-plane for (a) θ_i =0°, (b) θ_i =5°, and (c) θ_i =10°. (d) "Near-field cross-section" as defined by (10) normalized to the geometric cross-section of the cloaked object.

To quantify the performance of this cloak, we introduce the following figure of merit, which is based on the deviation of the electric field intensity from the uniform intensity E_0^2 of a plane wave,

$$\sigma_{NF} = \int (E^2 - E_0^2) dA \,/E_0^2, \qquad (10)$$

where integration is carried over a sphere surrounding the structure. The above quantity has the units of area, and therefore may be referred to as a "near-field cross-section" of the scattering structure, as opposed to the true scattering cross-section based on the far-fields. The plot of this visibility measure, which by no means is a unique possible choice, versus the incidence angle is shown in Fig.1d. We may conclude that the structure operates as a visibility-reducing device for incidence angles up to approximately 3 degrees; its visibility increases rapidly for larger angles.

References

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