Alternative material parameters for transformation optics designs

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Abstract

We present a method to find alternative/simplified material parameters for 2-D transformation optics (TO) devices that operate under a fixed illumination. The alternative material parameters support the same field pattern as the original material parameters of the TO device. Analytical calculations of the alternative parameters are shown and the results are verified through the full-wave simulations of a TO device: an electromagnetic field rotator. Although the TO devices possessing alternative material parameters only work for a particular illumination, the method presented will application in antennas and beam forming networks.

1. Introduction

A transformation optics device establishes a desired electromagnetic field pattern through an anisotropic and inhomogeneous distribution of material parameters [1-2]. In such a device, the material parameters as well as their spatial variation are defined through a coordinate transformation. The electromagnetic fields, wave vectors and Poynting vectors at every point in the transformed domain can be written in terms of the coordinate transformation [3].

We show that, in addition to the material parameters defined by a specific coordinate transformation, there exist infinitely many material parameter sets that will produce exactly the same field distribution. The downside, however, is that the devices with alternative material parameters only work for a particular illumination.

2. Concept

In order to easily describe how the alternative material parameters are found, we will restrict ourselves to 2-D transformation optics designs. Without loss of generality, let us further assume that the electromagnetic waves are vertically polarized: the only non-zero field quantities in the medium are the z component of electric field E_z and the H_x and H_y components of magnetic field. Therefore, the relevant material parameters become a 2×2 relative permeability tensor $\overline{\mu}$ in the x-y plane, and a scalar relative permittivity ε_z in the z direction. Under such conditions, Maxwell's time-harmonic, source-free equations for plane-wave propagation can be written as:

$$\text{(a)} \quad -j\vec{k}\times\vec{E} = -j\omega\overline{\overline{\mu}}\mu_o\vec{H} \qquad\qquad \text{(b)}\ -j\vec{k}\times\vec{H} = j\omega\varepsilon_z\varepsilon_o\vec{E} \qquad (1)$$

These two vector equations can be rewritten as a set of scalar equations:

$$\text{(a)} \quad \begin{pmatrix} \bar{k}_y \\ \bar{k}_x \end{pmatrix} = \begin{pmatrix} \mu_{xx} & -\mu_{xy} \\ -\mu_{xy} & \mu_{yy} \end{pmatrix} \begin{pmatrix} \bar{\eta}_y^{-1} \\ \bar{\eta}_x^{-1} \end{pmatrix} \qquad \text{(b)} \quad \varepsilon_z = \left(\frac{\bar{k}_x}{\bar{\eta}_x} + \frac{\bar{k}_y}{\bar{\eta}_y} \right) \tag{2}$$

where $\bar{\eta}_x, \bar{\eta}_y$ are the wave impedances and $\bar{k}_x, \bar{k}_y$ are the wavenumbers in the x and y directions, normalized with respect to their respective free space values.

Given that $\bar{k}_x, \bar{k}_y, \bar{\eta}_x, \bar{\eta}_y$ are known in a transformation optics device, the permeability and permittivity can be expressed in terms of these quantities using equation (2). Matrix equation (2a) is an underdetermined system since we have three unknowns (three permeability entries) and two equations. Therefore, an infinite number of solutions (permeability tensors) can be found to satisfy a set of wave numbers and wave impedances. Assuming that both wave impedances are finite, the solutions can be written as follows:

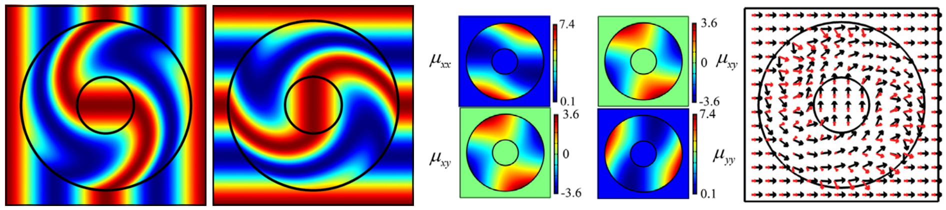
$$\bar{\bar{\mu}} = \begin{pmatrix} \bar{k}_y \bar{\eta}_y + \dfrac{\bar{\eta}_y}{\bar{\eta}_x} \mu_{xy} & \mu_{xy} \\ \mu_{xy} & \bar{k}_x \bar{\eta}_x + \dfrac{\bar{\eta}_x}{\bar{\eta}_y} \mu_{xy} \end{pmatrix} \qquad \varepsilon_z = \left(\frac{\bar{k}_x}{\bar{\eta}_x} + \frac{\bar{k}_y}{\bar{\eta}_y} \right), \tag{3}$$

where μ_{xy} can be chosen arbitrarily. In summary, (3) shows that if the wavenumbers and wave impedances are known in a medium, numerous alternative material parameters can be found.

This fact can be utilized to define alternative material parameters for a transformation optics design. If wavenumbers $(\bar{k}_x, \bar{k}_y)$ and wave impedances $(\bar{\eta}_x, \bar{\eta}_y)$ are known within the transformation optics device under a given plane-wave illumination, the original material parameters can be replaced by alternative ones calculated using (3). This replacement of material parameters can take place within any part or over the entire domain of the transformation optics device. It guarantees the same $\bar{k}_x, \bar{k}_y, \bar{\eta}_x, \bar{\eta}_y$ within the device, under the particular plane-wave excitation.

3. Verification

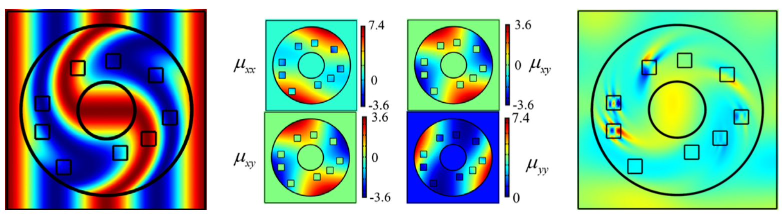
To verify the proposed method, we will consider an electromagnetic field rotator [4]. We compare the simulated performance of a field rotator implemented with the original material parameters defined by the coordinate transform in [4] and with alternative parameters. The simulations are performed using the finite element solver, Comsol Multiphysics. Fig. 1a and Fig. 1b depict the vertical (out of plane) electric field within and surrounding the field rotator when illuminated by a plane wave propagating in the x (horizontal) and y (vertical) directions, respectively. The field rotator implemented with the original material parameters shown in Fig. 1c, performs a 90 degree field rotation independent of excitation direction. Fig. 1d shows the wave vector and Poynting vector distribution within and surrounding the field rotator when it is illuminated by a plane wave propagating in x direction (see Fig. 1a). The Poynting vector is indicative of the wave impedances given that the ratio of wave impedances stipulates the Poynting vector direction.



(a) (b) (c) (d)

Fig. 1: A time snapshot of the vertical electric field within and surrounding the electromagnetic field rotator. The out of plane electric field is shown for plane-wave incidence along the x axis (a) and y axis (b). The material parameters for the device are shown in (c). Wave vector (red) and Poynting (black) vector directions within the device are shown in (d), for a plane wave illumination along the x-axis.

Now, the square patches in Fig. 2a are replaced with materials possessing alternative parameters. Within the square patches, the off-diagonal permeability entries are set to zero, and alternative material parameters (see Fig. 2b) are calculated using equation (3) and the vector distributions shown in Fig. 1d. For plane wave incidence along the x direction, the field rotator performs exactly as before (see Fig. 2a). However, since the alternative material parameters are found for a plane wave illumination along the x direction, the device does not work for other plane wave illuminations. Fig. 2c shows the field distribution for plane-wave incidence along the y direction.



(a) (b) (c)

Fig. 2: A time snapshot of the vertical electric field within and surrounding the field rotator implemented using alternative material parameters. The square patch regions are replaced with alternative material parameters. (a) plots the field distribution for a plane wave illumination along the x direction. (b) shows the alternative material parameter distributions. (c) shows that the field rotator does not work for a plane wave illumination along y direction.

Conclusion

In this paper, we presented a method to calculate alternative material parameters for transformation optics (TO) devices based on the wavenumber and wave impedance distributions within the devices. The method is verified through full wave simulation. The possibility of obtaining the same field distribution with different material parameter sets is clearly shown. The proposed method could be used to change the material parameters of TO devices to those that are simpler to fabricate.

References

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