New Transmission-line Unit Cells for Building Transformation Electromagnetics Devices

Michael Selvanayagam[1] **and George V. Eleftheriades**[1]

[1]Department of Electrical and Computer Engineering
University of Toronto
40 St. George Street, Toronto, Canada
email: gelefth@waves.utoronto.ca

Abstract

Transformation electromagnetics requires anisotropic metamaterials which work for multiple polarizations. These can be very difficult to realize using currently existing metamaterials. In this paper, two new metamaterial unit cells are introduced. One which can implement an anisotropic medium with a full tensor and another which supports two-polarizations. These unit cells can be quite useful for implementing various devices designed using transformation electromagnetics.

1. Introduction

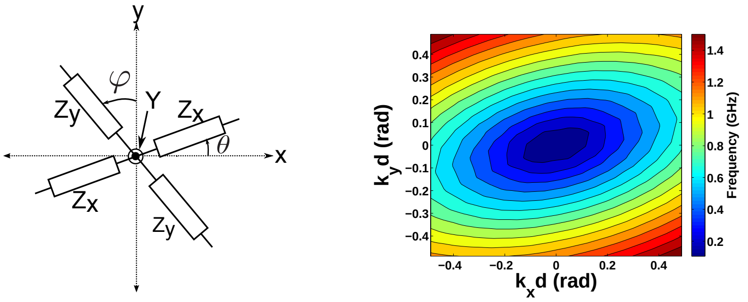
Transformation electromagnetics (TREM) has introduced a new way of shaping the propagation of electromagnetic fields. By applying a coordinate transform to Maxwell's equations, an equivalent set of material parameters can be found which physically implement this coordinate transform [1, 2]. By using this procedure, novel electromagnetic devices can be designed by determining an appropriate set of material parameters. This has introduced a class of new devices such as electromagnetic cloaks [3], as well as novel designs for many 'classical' devices such as lenses and radomes [4].

The material parameters found using transformation electromagnetics are, in general, inhomogeneous and anisotropic with off-diagonal components in the material tensor. The corresponding material parameters also require both an electric and a magnetic response, which leads to multi-polarization operation.

Currently, most experimental transformation electromagnetics devices respond to a single polarization. Moreover, to alleviate the complexity of transformation electromagnetic metamaterials, there have been attempts to minimize the anisotropy, either by relaxing the material parameters [3] or by using a conformal transformation [5]. Thus, to implement the required transformation electromagnetic material parameters has been a challenge. In this paper, two new ideas for metamaterial unit cells are introduced which demonstrate how to handle anisotropy and polarization dependence as required for transformation electromagnetics. This is done within the context of transmission-line metamaterials. The first idea introduces a skewed transmission-line grid to implement a material tensor with off-diagonal components. The second idea introduces a multi-layered volumetric unit cell which supports two different polarizations.

2. An Anisotropic Metamaterial With Off-diagonal Components

The discussion begins with the two-dimensional, shunt-node, transmission-line unit cell [6]. Generally, such a unit cell will have a scalar permitivitty, ϵ_z and at most an anisotropic permeability tensor, $\bar{\bar{\mu}} = \begin{bmatrix} \mu_{xx} & 0 \\ 0 & \mu_{yy} \end{bmatrix}$. To construct a unit cell that has an effective permeability tensor with off-diagonal components, the usually Cartesian transmission-line grid can be placed on a skewed-lattice as shown in Fig. 1a. By doing this, skew angles θ and φ are introduced to the unit cell which add extra degrees of freedom along with the series and shunt loadings. This allows the unit cell to implement an effective medium tensor with $\bar{\bar{\mu}} = \begin{bmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{xy} & \mu_{yy} \end{bmatrix}$.



(a) (b)

Fig. 1: (a) A skewed transmission-line grid. Z_x, Z_y and Y represent the series impedances and shunt admittance of the transmission-line grid respectively. The skew angles, θ and φ implement the skew of the transmission-line grid.(b) An example two-dimensional dispersion relation for the skewed transmission-line unit cell.

The dispersion relation of the unit cell is examined using full-wave simulation for specific values of $Z_x, Z_y, Y, \theta, \varphi$ and is shown in Fig. 1b. Here it can be seen that the isofrequency contours are elliptical, indicating anisotropy and are also tilted indicating that $\mu_{xy} \neq 0$.

The effective medium parameters for such a unit cell are described by the following equations

$$\mu_{xx} = \frac{1}{\cos(\theta - \varphi)} \left[Z_y \cos^2\theta + Z_x \sin^2\varphi \right], \qquad (1)$$

$$\mu_{yy} = \frac{1}{\cos(\theta - \varphi)} \left[Z_y \sin^2\theta + Z_x \cos^2\varphi \right], \qquad (2)$$

$$\mu_{xy} = \frac{1}{\cos(\theta - \varphi)} \left[Z_y \sin\theta\cos\theta - Z_x \sin\varphi\cos\varphi \right], \qquad (3)$$

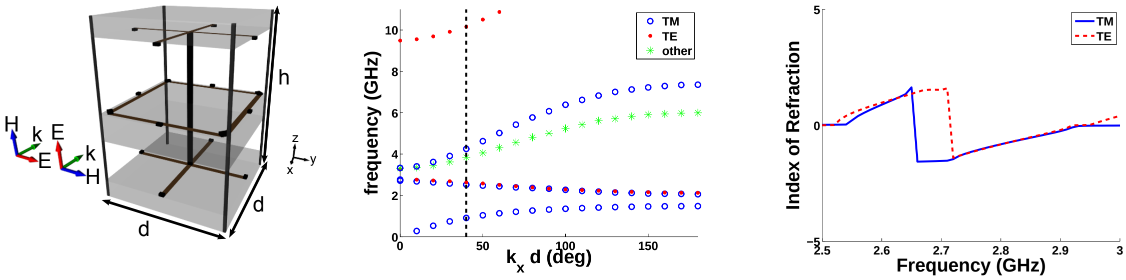
By introducing the skew angles θ and φ into the unit cell the material parameters also become a function of θ and φ, showing the effect of the skew angles on the material parameters.

While the example here has focused on a two-dimensional transmission-line grid, this skewed-lattice concept can be easily extended to a volumetric topology to implement full-tensor anisotropy for free-space propagation.

3. A Dual-Polarized Metamaterial

The other limitation of existing metamaterial unit cells is their response to a single polarization only. While multi-polarized unit cells have been proposed, they are often very complicated to fabricate, requiring for example, a three-dimensional assembly of printed circuit boards. Here a volumetric, transmission-line metamaterial that can handle two different polarizations and is simpler to fabricate is introduced.

The unit cell is able to support two different polarizations because it has two different transmission-line layers, a series-node layer and a shunt-node layer as shown in Fig. 2a. The series-node layer supports a polarization with a magnetic field perpendicular to the layer (TM) while the shunt-node layer supports a polarization with the electric field perpendicular to the layer (TE). This allows the unit cell to support any polarization for any incident cylindrical wave. Also note that the fabrication of a group of unit cells is relatively simple as it can be made by simply stacking layers of printed circuit boards.



(a) (b) (c)

Fig. 2: (a) A schematic of the dual-polarized unit cell. Note the different transmission-line layers, the series-node layer and the shunt-node layer. (b)A one-dimensional dispersion relation. Note the two overlapping backward-wave modes which correspond to the TE and TM polarizations. (c) The effective index of refraction, showing again, an identical negative index of refraction for both polarizations.

The dispersion relation of the unit cell is shown in Fig. 2b and it can be seen that the unit cell supports two backward-wave modes, a TE and a TM mode, showing it's dual-polarized nature. Likewise, the effective medium properties of the unit cell, specifically the index of refraction are shown in Fig. 2c. Here is is seen that the unit cell has an identical negative index of refraction for both the TE and TM polarizations.

Thus by using this configuration of unit cells, transformation electromagnetic devices which are polarization independent can be implemented.

4. Conclusion

Transmission-line metamaterial unit cells with either full-tensor anisotropy or dual-polarization have been introduced. As stated above, the former, while demonstrated for 2D-transmission line grids, can easily be extended to volumetric metamaterials by skewing the transmission lines found in the volumetric unit cell. By combining these two concepts together one can construct a volumetric unit cell which is both dual-polarized and anisotropic. Using this approach would allow one to implement a whole new class of transformation electromagnetics devices.

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