

# Active non-Foster Metamaterials: From Intriguing Background Physics to Real-world Application

S. Hrabar

Faculty of Electrical Engineering and Computing, University of Zagreb  
Unka 3, Zagreb, HR-10000, Croatia  
Fax: + 385-16129717; email: Silvio.Hrabar@fer.hr

## Abstract

This plenary talk reviews the basic concepts, counter-intuitive background physics, and foreseen applications of recently introduced active dispersionless non-Foster metamaterials.

## 1. Introduction

All known passive materials (or metamaterials) that have either negative (ENG, MNG) or less-than-unity (ENZ, MNZ) real parts of permittivity or permeability are always dispersive and suffer from inherent narrow operating bandwidth. This drawback is caused by the basic underlying physics of energy stored within passive loss-free material:

$$W = \frac{1}{2} \frac{\partial[\omega \cdot \varepsilon(\omega)]}{\partial \omega} |E|^2 + \frac{1}{2} \frac{\partial[\omega \cdot \mu(\omega)]}{\partial \omega} |H|^2. \quad (1)$$

Here,  $E$  is the electric field and  $H$  is the magnetic field. The net energy is a strictly positive quantity, thus  $W > 0$  in (1). In addition, some energy is necessary for polarizing any material, thus the energy in (1) is also always greater than the energy stored in vacuum ( $W_0$ ). This leads to well-known energy-dispersion constraints (the inequalities at the most right part of (2)):

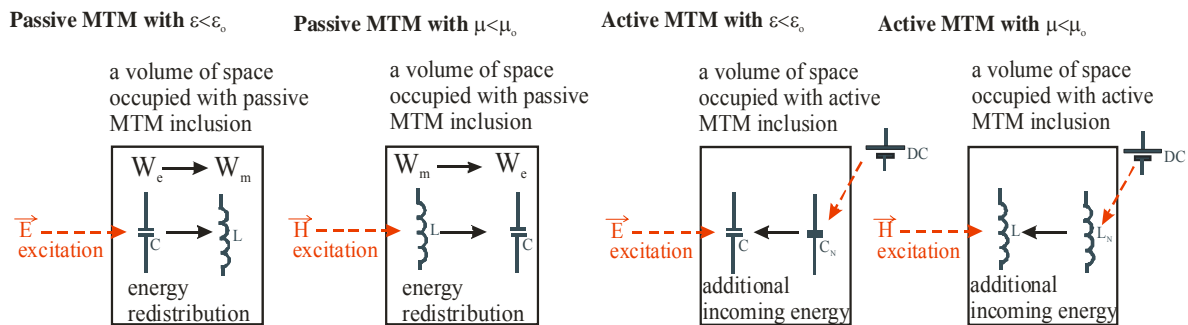
$$W_0 = \frac{1}{2} \varepsilon_0 |E|^2 + \frac{1}{2} \mu_0 |H|^2, \quad W > W_0 \quad \Rightarrow \quad \frac{\partial[\varepsilon(\omega)]}{\partial \omega} > 0, \quad \frac{\partial[\mu(\omega)]}{\partial \omega} > 0. \quad (2)$$

In (2),  $\varepsilon_0$  and  $\mu_0$  stand for free-space permittivity and permeability, respectively. From (2) one concludes that  $(\partial\varepsilon/\partial\omega) > 0$  and  $(\partial\mu/\partial\omega) > 0$  for *any* lossless passive material (or metamaterial). The circuit theory analogue of (2) is Foster's reactance theorem ( $(\partial X/\partial\omega) > 0$  and  $(\partial B/\partial\omega) > 0$ ,  $X$  and  $B$  being reactance and susceptance, respectively). In the case of 'negative' and 'less-than-unity', materials, the constraints in (2) actually show the existence of a resonant phenomenon, associated with energy redistribution from electric into magnetic field (ENZ or ENG case) or vice versa (MNZ or MNG case). This process is sketched in Fig. 1. It is important to stress that all known passive metamaterials (Split-Ring-Resonator-based, wire-based, Complementary-Split-Ring-Resonator-based, transmission-line based, 'fishnet'-based etc.) behave in the same way, and therefore suffer from dispersion.

## 2. Basic Physics of Non-Foster-based Metamaterials

There are electronic circuits that behave as negative capacitors or negative inductors [3] and violate the energy-dispersion constraints and Foster theorem (so-called 'non-Foster' elements). Negative capacitors and negative inductors have dispersion curves that are the exact inverse of the dispersion curves of ordinary 'positive' elements. Therefore, one could expect that the dispersion of ordinary

passive metamaterials can be compensated for with the ‘inverse’ dispersion of non-Foster elements, resulting in broadband behavior [4-11]. The basic idea is sketched in Fig.2.



**Fig. 1:** The basic principle of passive metamaterial **Fig. 2:** The basic principle of active non-Foster metamaterial

Instead of energy redistribution from the electric field into magnetic field (or vice versa) used in passive metamaterial, here one introduces an additional energy flow from active device (that has its own DC power supply). For instance, in the case of ENZ non-Foster metamaterial [5] one applies the parallel combination of a (positive) distributed capacitance ( $C$ ) (of some host transmission line) and a negative capacitor ( $C_N$ ). In this way, the overall capacitance is decreased below the free-space value yielding frequency-independent ENZ behavior [5]:

$$\epsilon_r(\omega) = [C / \epsilon_0 - |C_N| / (\epsilon_0 \Delta)]. \quad (3)$$

Here  $\Delta$  stands for the unit cell dimension. It is important to understand this counter-intuitive behaviour. The negative capacitor behaves as a frequency-dependent active inductor (a source) that supplies additional current to the positive capacitor. This additional current causes faster charging and therefore decreases effective capacitance. Of course, similar principle can be used for MNZ metamaterials. In that case, one should have a series combination of distributed inductance  $L$  and negative inductor  $L_N$  (Fig. 2) that would lower effective relative permeability below the free-space value:

$$\mu_r(\omega) = [L / \mu_0 - |L_N| / (\mu_0 \Delta)]. \quad (4)$$

### 3. State of the Art, Possible Applications and Future Trends

The ‘non-Foster’ networks are based on Negative Impedance Converter, originally introduced back in the 1950’s [3]. Although this idea is indeed old, there are only a few papers in the open literature that report successful implementations of negative capacitors or inductors due to serious stability problems [9]. In our recent study [10] it was shown that the stability criteria are strongly dependent on the configuration of a passive network connected to the non-Foster element. Thus, assuring stability of non-Foster circuit is rather difficult task, but it is certainly feasible (for some predetermined topology of load network, as detailed in [10]). The first non-Foster-based active dispersionless ENZ metamaterials was presented in [5] and further improved in [6,7]. The negative capacitors were constructed using FET-based circuits incorporated into 2D microstrip unit cells. Measurement of equivalent effective permittivity showed fairly constant ENZ behaviour ( $0.25 < \epsilon_r < 0.35$ ) within one octave (1GHz-2GHz) [6]. These measured results were also used as input data for ADS<sup>TM</sup> simulation of proposed active 2D plasmonic cloak [7] and showed operating relative bandwidth of 100% (comparing to relative bandwidth of 20% of passive cloak). In the subsequent study [8] a complete 1D active RF ENZ metamaterial (based on air transmission line loaded with three op-amp-based negative capacitors) was analysed, built and tested. Measured real part of effective permittivity was rather constant (it varied from 0.27 to 0.37), in the frequency range 2 MHz to 40 MHz (bandwidth of more than four octaves). This bandwidth is significantly wider than a bandwidth of any passive ENZ metamaterial. In addition, it was found that this type of line supports counter-intuitive (but causal) superluminal phase

and group velocities [11]. The improved version of this metamaterial, which will be presented at the conference, operates in the frequency range 4 MHz – 224 MHz (a bandwidth of more than six octaves).

In order to extend described principles to ‘volumetric’ ENZ metamaterial, one might use an array of the short dipoles loaded with negative capacitors. We developed the 2x2 array of dipoles in low RF frequency range (up to 50 MHz), which was mounted within a parallel-plate capacitor [12]). Achieved bandwidth (one octave) is again wider than a bandwidth of any passive ENZ metamaterial, but, it is clearly narrower than the bandwidth of 1D active TL-based ENZ metamaterial reported in [8]. It was found that this happens due to capacitive coupling between neighboring dipoles. This coupling cannot be compensated for by non-Foster elements (in the case of TL metamaterial in [8], the ‘inclusions’ are tightly coupled by direct connection to a host transmission line). Similar approach was used for construction of volumetric MNZ metamaterials [12]. Proposed volumetric active ENZ and MNZ metamaterials might find application in ultra-broadband active antennas [12].

Obviously, it would be very convenient to use TL-based non-Foster metamaterials (with excellent dispersionless properties [8]) as a part of volumetric structures. This is the second approach, currently being investigated in our group. It is based on 2D non-Foster TL interfaced with free-space via ‘transition’ layer [12]. This approach would be convenient for application in cloaking technology as originally suggested in [6]. Our first simulations [12] indicated that should be possible to obtain a behavior very similar to the behavior of hypothetical volumetric continuous material within the bandwidth of at least two octaves. Experimental verification of this approach is in progress and the first results will be reported at the conference.

One might argue that the all presented prototypes operate in RF frequency region and that it might be difficult to bring this concept into the microwave region. Our preliminary numerical study showed that using standard 130 nm CMOS technology it should be possible to increase the highest operating frequency of non-Foster element (and associated non-Foster metamaterial) to above 5 GHz.

#### 4. Conclusion

A concept of dispersionless active non-Foster metamaterial was reviewed and a couple of representative examples were given. It was shown that this approach enables construction of ultra-broad-band RF devices, which might find application in antennas, communications, and cloaking technology.

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