Effective properties of a two-dimensional magnonic metamaterial

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Abstract

In this paper the metamaterial properties of a two-dimensional magnonic crystal are studied. The system is composed by holes embedded into a Permalloy ferromagnetic film. Both the periodicity of the magnonic crystal and the diameter of the holes are in the nanometric range. Since holes are much smaller than the characteristic mode wavelength commensurable with the periodicity, the dynamics is described in terms of effective quantities. Due to this condition also the characteristic wavelength is defined as an effective wavelength. It is shown that the effective wavelength is not necessarily equal to the well-known Bloch wavelength, because of the finite size of the holes.

1. Introduction

In these last years great attention has been given to the study of metamaterials both for their interesting properties and for the high number of potential applications. Among them two interesting classes of metamaterials are electromagnetic and plasmonic metamaterials. Indeed, since the wavelength of electromagnetic waves and of plasmonic waves is much larger than the periodicity of the artificial arrays [1], the description of the most important physical quantities can be made in terms of effective properties.

The aim of this paper is to describe in terms of effective properties and to classify as a metamaterial a special kind of magnonic crystal (MC) represented by a periodic two-dimensional (2D) array of antidots (ADs) (holes) of diameter δ embedded into a continuous film of Permalloy ($Ni_{80}Fe_{20}$, Py). Calculations were performed by means of a micromagnetic approach based on the dynamical matrix method (DMM) [2]. From the inspection of spatial profiles of collective modes it is possible to identify a characteristic wavelength which is commensurable with the periodicity a of the system. Since collective modes are mainly affected by the finite size of the holes rather than the periodicity and since the characteristic wavelength is much larger than δ, the dynamics is described in terms of effective properties. These properties are those that characterize the 2D array of ADs as a metamaterial. In this way, the characteristic wavelength can be regarded as an effective wavelength λ_{eff}. Interestingly, the effective wavelength, which can be defined for each mode of the spectrum, is not necessarily equal to the Bloch wavelength and, in the geometry studied, assumes at most a value equal to $2a$. In the special case studied (δ = 120 nm and a = 800 nm) the ratio $\delta/\lambda_{\mathrm{eff}} \approx 0.1 \ll 1$ for the whole range of Bloch wave vectors investigated. The condition $\delta/\lambda \ll 1$ is not always fulfilled by replacing λ_{eff} with the Bloch wavelength λ_B. Correspondingly, also an effective wave vector is introduced.

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2. Discussion and results

In Fig.1the Damon-Eshbach (DE) geometry studied at $\Phi = 0$ (angle between $\boldsymbol{H}$ and the y-axis) is also depicted. In this geometry the external field $\boldsymbol{H}$ is placed along the y-axis, while the Bloch wave vector $\boldsymbol{K}$ is parallel to the x-axis. In order to highlight the metamaterial properties of this special kind of MC, we restrict ourselves to the stationary regime present at the Brillouin zones (nBZs) edges with n=1,2,...

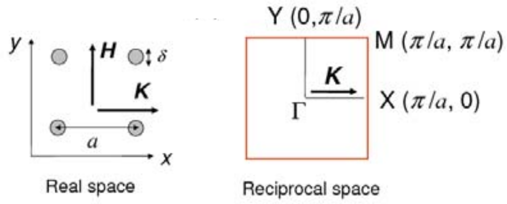

Fig.1. The configuration in the real space is depicted: the Bloch wave vector $\boldsymbol{K}$ is perpendicular to $\boldsymbol{H}$ in the AD x-y plane (DE geometry). The 1BZ in the reciprocal space with the high-symmetry points and the high-symmetry direction ΓX are also shown.

Since we deal with a periodic system, collective excitations are supposed to fulfil the Bloch rule, namely $\delta\boldsymbol{m}(\boldsymbol{r}+\boldsymbol{R}) = \delta\boldsymbol{m}(\boldsymbol{r})\, e^{i\boldsymbol{K}\cdot\boldsymbol{R}}$ where $\delta\boldsymbol{m}(\boldsymbol{r})$ is the dynamic magnetization and $\boldsymbol{R}$ is a lattice vector. By inspecting spatial profiles of collective modes, we define an effective wavelength λ_{eff} as the distance between two maxima (or two minima) corresponding to the effective periodicity of the wave. This effective wavelength is directly related to the scattering of magnonic modes on holes and does not fulfil the rules of Bloch periodicity. Hence, it is not necessarily equal to the Bloch wavelength λ_{B}. As an example, we show in Fig.2 the spatial profiles of two modes at the border of the 3BZ belonging to the family of localized modes, called $\mathrm{DE}_{3\mathrm{BZ}}^{\mathrm{loc}}$ and $\mathrm{DE}_{4\mathrm{BZ}}^{\mathrm{loc}}$. Here, the superscript "loc" means localized along the horizontal rows of ADs. For this couple of modes $\lambda_{\mathrm{eff}}^{3\mathrm{BZ}} = 2a$ and $\lambda_{\mathrm{B}} = 2/3a$ as indicated, that is the effective wavelength is three times larger than the Bloch wavelength .

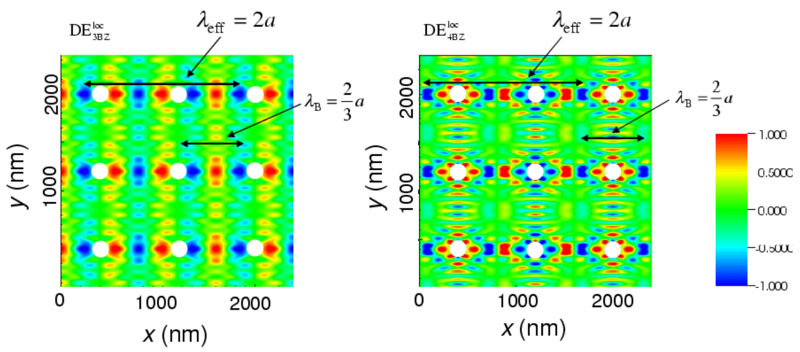

Fig.2. Calculated spatial profiles of $\mathrm{DE}_{3\mathrm{BZ}}^{\mathrm{loc}}$ and $\mathrm{DE}_{4\mathrm{BZ}}^{\mathrm{loc}}$ modes in 3×3 primitive cells according to DMM. The effective and the Bloch wavelengths are indicated.

Owing to the above considerations, it is natural to define also a corresponding effective wave vector $\boldsymbol{k}_{\mathrm{eff}}$. The following general rules can be established:

1) The effective wavelength is commensurable with the periodicity of the sample. This means that

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$$\lambda_{\text{eff}}^{n\text{BZ}} = \begin{cases} 2a \text{ if } n \text{ is odd} \\ a \quad \text{if } n \text{ is even} \end{cases} \tag{1}$$

Apart from the Γ point where it is infinite, the effective wavelength assumes alternatively the values $2a$ and a.

2) At the border of the 1BZ and at the equivalent points that differ by multiple integers of **G** = $(2\pi/a, 0)$, the effective wave vector assumes the same value, viz.

$$\boldsymbol{k}_{\text{eff}}\left[\boldsymbol{K} = \left(\frac{\pi}{a}(1+2m),0\right)\right] = \left(\frac{\pi}{a},0\right) \tag{2}$$

with m =0,1,2,….

3) At the border of the 2BZ and at the equivalent points that differ by multiple integers of **G** = $(2\pi/a, 0)$, the effective wave vector assumes the same value, viz.

$$\boldsymbol{k}_{\text{eff}}\left[\boldsymbol{K} = \left(\frac{2\pi}{a}(1+m),0\right)\right] = \left(\frac{2\pi}{a},0\right) \tag{3}$$

with m = 0,1,2,…
In a compact form a simple relation between the effective wave vector and the Bloch wave vector in the reduced zone scheme can be written at the edge of the given nBZ with n= 2,3,.., namely

$$\boldsymbol{k}_{\text{eff}}^{n\text{BZ}} = \boldsymbol{K}^{1\text{BZ}} + h\boldsymbol{G}/2 \text{ with } \begin{cases} h=0 \quad \text{for} \quad n \text{ odd} \\ h=1 \quad \text{for} \quad n \text{ even} \end{cases} \tag{4}$$

where $\boldsymbol{K}^{1\text{BZ}} = (\pi/a, 0)$ is the Bloch wave vector at the edge of the 1BZ and $\boldsymbol{G} = (2\pi/a)$. Of course, at the edge of the 1BZ $\boldsymbol{k}_{\text{eff}}^{1\text{BZ}} = \boldsymbol{K}^{1\text{BZ}}$. Interestingly, in the limit for $\delta \to 0$, we have found that the effective wavelength becomes equal to the Bloch wavelength and the description in terms of effective properties breaks down.
A magnonic device consisting of the array of ADs and of a light scattering apparatus can be built. Taking into account the relation between the transferred light wave vector $\boldsymbol{q}$ and the Bloch wave vector $\boldsymbol{K}$ of magnonic modes, namely $\boldsymbol{q} = \boldsymbol{K}$, and (4), the "resonance" condition between light and magnetic material can be written in terms of the effective wave vector. Information about effective dynamic properties can be extracted from the experiment.

4. Conclusion

In this work the metamaterial properties of a 2D array of ADs were investigated by defining some effective quantities characterizing the magnonic mode dynamics. Due to their finite but small size, holes rather than periodicity affect the dynamics. It was shown that a characteristic wavelength much larger than the hole size can be regarded as an effective wavelength. The effective wavelength can be either equal or twice the periodicity at the edge of BZs. A relation between the corresponding effective wave vector and the Bloch wave vector was obtained. These properties were found in the DE geometry and for the external field $\boldsymbol{H}$ applied along the y-direction, but can be easily generalized to other geometries. A simple application based on these metamaterial properties was also given.

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