Multi-layer fishnet metamaterials as magnetic hyperbolic media

S. S. Kruk, D. A. Powell, A. Minovich, D. N. Neshev, and Yu. S. Kivshar

Nonlinear Physics Centre, Research School of Physics and Engineering
Centre for Ultrahigh-bandwidth Devices for Optical Systems (CUDOS),
The Australian National University, ACT 0200, Canberra, Australia
Fax: +61-2-6125-8588; email: ssk124@physics.anu.edu.au

Abstract

We study the anisotropic properties of multi-layer fishnet metamaterials and reveal that such structures allow the realization of generalized indefinite media with hyperbolic dispersion. In contrast to other hyperbolic media, multi-layer fishnet metamaterials may have not only effective permittivity tensor $\hat{\varepsilon}$ but also effective permeability tensor $\hat{\mu}$ with negative components.

1. Introduction

Indefinite media proposed in Ref. [1] are artificial metamaterial structures which support propagating waves with extremely large wavevectors and density of states. Such media are described by the effective tensors of electric and/or magnetic susceptibilities where some of the components are negative, and the corresponding dispersion is *hyperbolic*. Importantly, such hyperbolic media are not only a hypothetical idea but can be practically realized in layered media. In the simplest case, it appears as an effective medium in the theories describing the averaged characteristics of the layered structure composed of alternating dielectric and metallic layers, a mesh created by metallic wires, or a plasmonic crystal of nanorods [2, 3, 4]. The presence of such hyperbolic dispersion opens a number of novel applications, including sub-wavelength imaging, sub-wavelength cavities, spontaneous emission control, thermal superconductivity, etc. At present such properties were discussed in metamaterials having only anisotropic $\hat{\varepsilon}$, while assuming $\mu = 1$.

In order to achieve a highly anisotropic $\hat{\mu}$-tensor, here we suggest utilizing multi-layer optical fishnet metamaterials with artificial magnetism. We choose these metamaterials [5] due to their low losses, bulk properties and double-negative response in wide spectral region. By studying the optical properties of the fishnet structures for the oblique light incidence and different polarizations, we derive their dispersion relations and show that these relations exhibit hyperbolic dispersion in the negative index spectral range. Our results shed new light on the optical emission in fishnet metamaterials and open new possibilities for imaging and emission control.

2. Media with indefinite permittivity and permeability

As discussed in Ref. [1], when considering general anisotropic media, with permittivity and permeability tensors given by:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \qquad \hat{\mu} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix}, \tag{1}$$

ISBN 978-952-67611-2-1

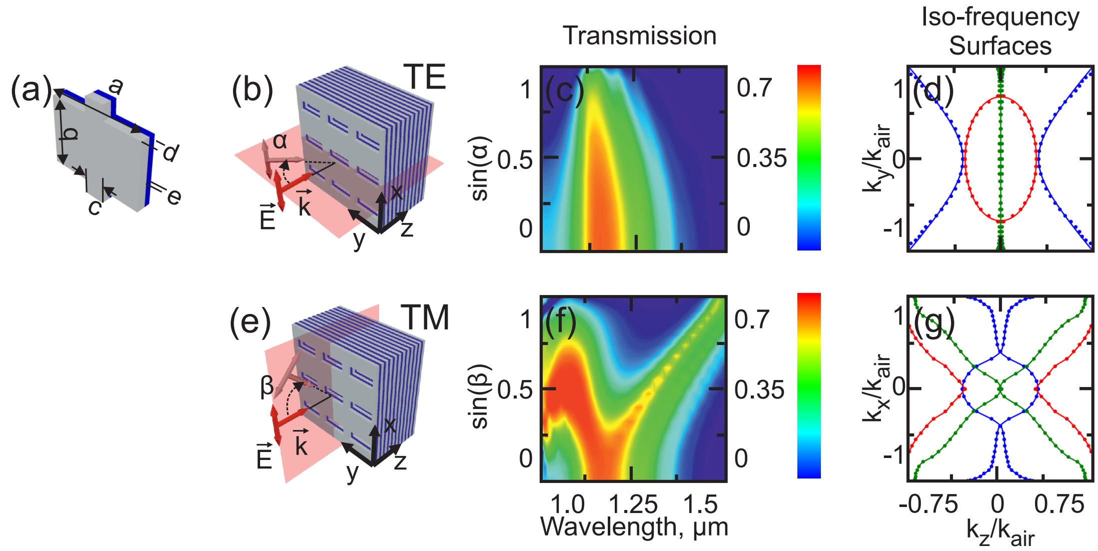

Fig. 1: (a) A unit cell for the fishnet metamaterial with design parameters $a = 500\,\text{nm}$, $b = 351\,\text{nm}$, $c = 100\,\text{nm}$, $d = 45\,\text{nm}$, $e = 30\,\text{nm}$. (b,e) TE and TM polarization of incident wave. (c,f) Transmission of ten-layers fishnet versus wavelength and angle of incidence for TE and TM. (d,g) Iso-frequency surfaces in k-space for wavelengths $1.03\,\mu\text{m}$ (red curves), $1.09\,\mu\text{m}$ (green curve) and $1.17\,\mu\text{m}$ (blue curve) for TE and TM polarizations.

there are several different regimes depending on the signs of the tensors components which appear in the dispersion relations for the z-propagation of both TE and TM waves:

$$\text{TE:} \quad \frac{k_y^2}{\varepsilon_x \mu_z} + \frac{k_z^2}{\varepsilon_x \mu_y} = \frac{\omega^2}{c^2}; \qquad \text{TM:} \quad \frac{k_x^2}{\varepsilon_z \mu_y} + \frac{k_z^2}{\varepsilon_x \mu_y} = \frac{\omega^2}{c^2}. \tag{2}$$

Anisotropic media with one or more negative components are called indefinite. To date, all indefinite media reported in the literature have either $\varepsilon_z < 0$ or $\varepsilon_{x,y} < 0$. This results in hyperbolic dispersion for TM polarized waves. However, this is only one possible case of indefinite media. In general, if both the permittivity and permeability have negative terms, then hyperbolic dispersion will occur for both TE and TM polarizations and will coincide with the negative phase velocity regime. We show below that this important case can be realized in multi-layer fishnet metamaterials.

3. Angular dispersion of multi-layer fishnets

Figure 1(a) shows the unit cell of our fishnet structure, which is similar to the unit cell of the double-layer fishnets proposed in Ref. [6] and is resonant in the near infra-red spectral region. The unit cell consists of metal (Ag) and dielectric (MgF_2) layers. At normal incidence, the transmission properties of the fishnet change dramatically with the number of layers. However, as the number of functional layers increases above approximately ten functional layers, the metamaterial properties remain practically constant. Figs. 1(b,e) show the geometry of the studied ten functional layers fishnet structures. We obtain the complex transmission and reflection spectra of the metamaterials numerically using CST Microwave Studio. For the permittivity of Ag we use the experimental data from Ref. [7], while for the MgF_2 we use a constant permittivity of $\varepsilon = 1.90$. In our numerical calculations we fix the electric field of the incident wave in the $x - z$-plane and consider the two cases of TE and TM polarizations as shown in Figs. 1(b,e).

Figs. 1(c,f) show the transmission of the ten functional layers fishnet versus wavelength and angle of incidence for the TE and TM cases, respectively. As one can see, for the TE case the transmission practically constant with changing angle of incidence. In contrast, for the TM case the transmission changes dramatically, showing the splitting of the transmission peak into two resonant modes. Similar

ISBN 978-952-67611-2-1

effect was previously observed in the case of single functional layer fishnet [8] and is closely related to the change of the magnitude of the incident electric filed across the gap between the metal layers.

Next, we extract the wavevector in the structure and obtain the dispersion of iso-frequencies. The boundary conditions ensure that $k_x = k_x^{air}$ and $k_y = k_y^{air}$. For the k_z component we use the inverted Fresnel formula [9]:

$$k_z = \pm\frac{1}{h}\cos^{-1}\left(\frac{1 - r^2 + t^2}{2t}\right) + \frac{2\pi m}{h} \tag{3}$$

Figure. 1(d) shows iso-frequency dispersion curves for the TE polarization. At wavelength 1.03μm (red curve) $Re[k_z] > 0$ and structure exhibits ordinary elliptic dispersion. At wavelength 1.17μm (blue curve) $Re[k_z] < 0$ and the structure exhibits hyperbolic dispersion [1]. At wavelength 1.09μm (green curve) $Re[k_z] = 0$ and the structure is in topological transition between elliptic and hyperbolic dispersions [10]. In Fig. 1(g) we also plot the iso-frequency contours corresponding to the TM case. In comparison to the TE case, we see that the dispersion is more complex. Interestingly, in all these cases the contours are diverging for larger k_y, indicating that this media should exhibit number of indefinite medium properties, such as high density of states and strong Purcell factor enhancement.

4. Conclusions

We have analyzed the dispersion properties of multi-layer fishnet metamaterials and revealed that such structures can be employed as generalized indefinite media with both anisotropic $\hat{\varepsilon}$ and $\hat{\mu}$ tensors having negative components. We have observed elliptic isofrequency surfaces as for the case of conventional anisotropic materials, hyperbolic isofrequency surfaces, which also are known to appear metal-dielectric layered metamaterials and wired metamaterials, and a number of exotic isofrequency contours for the case of plasmon excitation by the tilted electric field. The elliptic and hyperbolic dispersion regimes can be modeled by tensors $\hat{\varepsilon}$ and $\hat{\mu}$ with elements having only frequency dependence, but in most general case these tensors are $\vec{k}$ dependent, and the effects of nonlocality are important.

References

[1] D.R. Smith and D. Schurig, "Electromagnetic wave propagation in media with indefinite permittivity and permeability tensors," Phys. Rev. Lett. **97**, 077405 (2003).

[2] P. A. Belov and Y. Hao, "Subwavelength imaging at optical frequencies using a transmission device formed by a periodic layered metal-dielectric structure operating in the canalization regime" Phys. Rev. B **73**, 113110 (2006).

[3] A. Salandrino and N. Engheta, "Far-field subdiffraction optical microscopy using metamaterial crystals: Theory and simulations", Phys. Rev. B **74**, 075103 (2006).

[4] M. A. Noginov, Y. A. Barnakov, G. Zhu, T. Tumkur, H. Li, and E. E. Narimanov, "Bulk photonic metamaterial with hyperbolic dispersion", Appl. Phys. Lett. **94**, 151105 (2009).

[5] J. Valentine, S. Zhang, T. Zentgraf, E. Ulin-Avila, D. A. Genov, G. Bartal, and X. Zhang, "Three-dimensional optical metamaterial with a negative refractive index," Nature **455**, 376–U32 (2008).

[6] G. Dolling, M. Wegener, C. M. Soukoulis, and S. Linden, "Design-related losses of double-fishnet negative-index photonic metamaterials," Optics Express **15**, 11536–11541 (2007).

[7] E. D. Palik, *Handbook of Optical Constants of Solids* (Academic Press, 1985).

[8] A. Minovich, D. N. Neshev, D. A. Powell, I. V. Shadrivov, M. Lapine, I. McKerracher, H. T. Hattori, H. H. Tan, C. Jagadish, and Y. S. Kivshar, "Tilted response of fishnet metamaterials at near-infrared optical wavelengths," Phys. Rev. B **81**, 115109 (2010).

[9] C. Menzel, C. Rockstuhl, T. Paul, F. Lederer, and T. Pertsch, "Retrieving effective parameters for metamaterials at oblique incidence," Phys. Rev. B **77**, 195328 (2008).

[10] H.N.S. Krishnamoorthy, Z. Jacob, E. Narimanov, I. Kretzschmar, and V.M. Menon, "Topological transitions in metamaterials," arxiv 1109.5637 (2011).