Hyperbolic Metamaterials Realized with Two-Dimensional Transmission Lines

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Abstract

We demonstrate the realization of a metamaterial medium with the hyperbolic isofrequency surfaces in the wavevector space as a two-dimensional grid of transmission lines. The peculiar character of wave propagation in such a hyperbolic medium is visualized by the study of the cross-like emission pattern of a current source. Our results are supported by the direct solution of the Kirchhoff equations and an analytical theory.

1. Introduction

Hyperbolic medium is a uniaxial system, where the transverse $\varepsilon_{xx} = \varepsilon_{yy} \equiv \varepsilon_\perp$ and longitudinal $\varepsilon_{zz} = \varepsilon_\parallel$ dielectric constants have opposite signs [1]-[2]. Due to the hyperbolic isofrequency contours in wavevector space this medium exhibits a number of unusual properties. First, the waves at its boundary may exhibit negative refraction, similarly to the case of double-negative materials [3]. Second, the diverging density of photonic promotes ultra-high spontaneous emission rates [4]-[7]. This makes a concept of hyperbolic medium very promising for the broad-band tailoring of light-matter coupling and explains the ongoing intensive attempts to realize hyperbolic plasmonic metamaterials [8]. Still, that only truly conclusive experimental report of the hyperbolic medium we are aware of is restricted to the magnetized plasma in the microwave range [9].

The goal of this work is to investigate another opportunity to create the hyperbolic metamaterial by using transmission lines (TL). The TL approach to synthesis and design of metamaterials has been recently developed [10]-[12]. One, two and three-dimensional TLmetamaterials were introduced exhibiting both negative and positive effective material parameters. Nevertheless metamaterial TL revealed uniaxial effective material parameters have not been yet presented. Here we extend the TL approach to design and synthesis of two-dimensional hyperbolic metamaterials.

2. Hyperbolic Medium Design

We consider an anisotropic hyperbolic medium with the following material parameters:

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & 0 \\ 0 & \varepsilon_{yy} \end{pmatrix}, \quad \mu = \mu_z \tag{1}$$

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where $\varepsilon_{xx} > 0$, $\varepsilon_{yy} < 0$ and $\mu > 0$. For a TM-polarized wave (wave propagates in the xy plane, with the electric field polarized along the z axis) the dispersion relation has the familiar form:

$$\frac{(k_x)^2}{\omega^2 \varepsilon_{yy} \mu_z} + \frac{(k_y)^2}{\omega^2 \varepsilon_{xx} \mu_z} = 1 \tag{2}$$

where k_x and k_y are the x and y components of the propagation vector and ω is the frequency. It is clear seen that for such medium the isofrequency contour is a hyperbola due to different signs of the permittivities.

A unit cell of the two-dimensional TL metamaterial possessing 2×2 permittivity tensor $\overline{\overline{\varepsilon}}$ and scalar permeability μ_z is schematically shown in Fig. 1(a). For the case of generality the loads are represented as series admittances Y_x and Y_y and shunt impedance Z. First, we derive the dispersion equation for a two-dimensional structure consisting of unit cells [see Fig. 1(a)]. Such approximation is possible at low frequencies, when the radiation effects can be neglected. It can be done by summing up all the currents flowing to the node (x,y) and equating this sum to the ground (via impedance Z). The result is:

$$Y_x\left(U_{x-1,y} + U_{x+1,y}\right) + Y_y\left(U_{x,y-1} + U_{x,y+1}\right) - 2U_{x,y}\left(Y_x + Y_y\right) = U_{x,y}/Z \tag{3}$$

In the periodic structure we can look for a Bloch solution of the form $U_{x,y} = U_0 \exp(j(k_x d + k_y d))$ and transform Eq. (3) to

$$Y_x \sin^2(k_x d/2) + Y_y \sin^2(k_y d/2) = -1/(4Z) \tag{4}$$

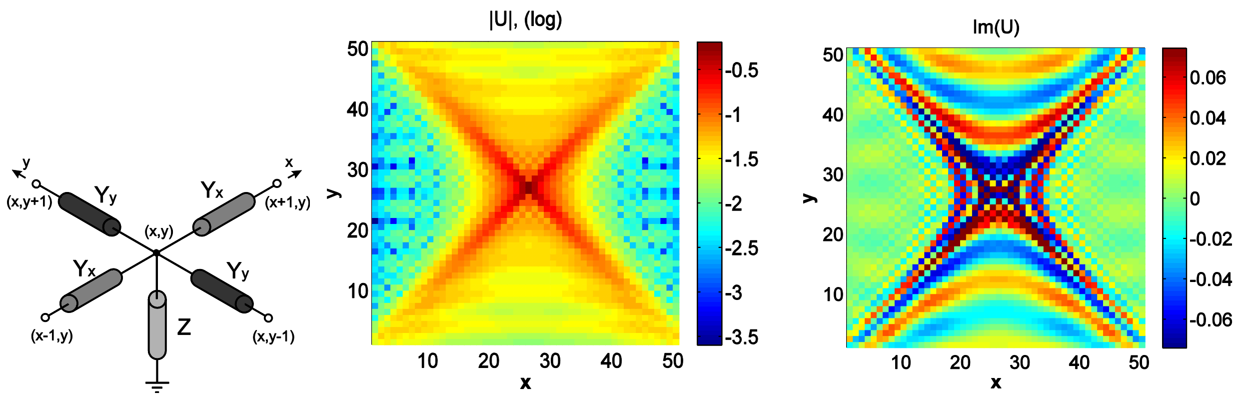
At the frequencies where the delays per unit cell are small ($k_x d \ll 1$, $k_y d \ll 1$) the dispersion relation (4) can be further reduced to:

$$-Y_x Z (k_x d)^2 - Y_y Z (k_y d)^2 = 1 \tag{5}$$

From comparison of (2) and (5) one may conclude:

$$Y_x / Y_y = \varepsilon_{xx} / \varepsilon_{yy} \tag{6}$$

From Eq. (6) it is clearly seen that the series admittances Y_x and Y_y should have opposite signs to provide hyperbolic shape of the dispersion curve. It is also possible to find the relation between material



(a) (b) (c)

Fig.1:The unit cell of two-dimensional metamaterial TL structure (a). The simulated voltage distribution of the two-dimensional hyperbolic medium composed of 51 × 51 unit cells: (b) the voltage magnitude in logarithmic scale; (c) the imaginary part of the voltage.

parameters of the medium and its two-dimensional transmission-line analogue:

$$\begin{aligned} j\omega\varepsilon_{yy}d &\to Y_x \\ j\omega\varepsilon_{xx}d &\to Y_y \\ 1/\left(j\omega\mu_z d\right) &\to Z \end{aligned} \tag{7}$$

The analysis described above is used to simulate the hyperbolic medium with material parameters:

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{pmatrix} 4\cdot 10^4 & 0 \\ 0 & -4\cdot 10^4 \end{pmatrix}, \quad \mu = 0.840\cdot\mu_0 \tag{8}$$

To simplify the selection of chip capacitors and inductors for fabrication of the hyperbolic medium sample the operational frequency is chosen to be 50 MHz. . The unit cells of the medium are assumed to have a cell dimension of d = 9 mm. Using (8), (7) we find the values of admittances and impedances of the 2-D unit cell as $Y_x = -j$, $Y_y = j$, and $Z=3j$. Now assuming the $Y=j\omega C$ and $Z=j\omega L$ the values of lumped elements in the two-dimensional TL unit cell can be found as $C_y=3.2$ nF, $C_x=-3.2$ nF and L=9.5 nH. At the frequency of operation the negative capacitance Cx can be equivalently represented as inductance Lx=3.2 nH. At the edges the structure is loaded by resistors R=1 Ohm to improve the boundary matching conditions.

Numerically simulated voltage distribution, excited in the structure composed of 51 × 51 unit cells, is shown in Fig. 1 (b). The current source is placed in the structure centre between two neighbouring nodes in y direction. The maximum of voltage distribution across the medium has the characteristic cross shape. In agreement with Eq. (5) the current flows in y direction. The oscillations of the imaginary part of the voltage, shown on Fig. 1 (c), also confirm the propagation of the emitted waves in y direction and their evanescent character along x axis.

3. Conclusion

We have designed and theoretically analyzed the two-dimensional hyperbolic medium based on a two-dimensional lattice of metamaterial TLs. We have calculated the structure parameters, including capacitances, inductances and matching condition, and observed the pronounced cross-like emission pattern of the current source, being the fingerprint of a hyperbolic medium. We have presented an analytical theory and also discussed the correspondence between the TLs and optical metamaterials. The fabrication and experimental studies of the hyperbolic media based on TLs is in progress.

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